Math 132 - November 11, 2016

Section 11.6: Absolute Convergence

- Definition: \( \sum a_n \) is absolutely convergent if \( \sum |a_n| \) converges.
- Definition: \( \sum a_n \) is conditionally convergent if \( \sum a_n \) converges but \( \sum |a_n| \) diverges.
- Theorem: If \( \sum a_n \) is absolutely convergent then the series is convergent.

Section 11.5: Alternating Series

Let \( S = \sum (-1)^n b_n \) where \( b_n > 0 \) be an alternating series. \( S \) converges if the following are true:

- \( b_{n+1} < b_n \) (so the \( b_n \) are decreasing)
- \( \lim b_n = 0 \)

And \( |R_n| = |S - S_n| \leq b_{n+1} \)

Warm-up Problems

1. **Clicker** Does the series converge absolutely: \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 + 2n + 1} \)
   - (a) Yes (b) No (c) It converges conditionally (d) IDK but I’m moving to Canada with Amy Schumer

Class Problems

2. **Alternating Series**: Does the series converge or diverge:
   \[
   \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} = 1 - \frac{1}{2^3} + \frac{1}{3^3} - \frac{1}{4^3} + \frac{1}{5^3} + \cdots
   \]

3. Are the following absolutely convergent? Are they conditionally convergent.
   
   - (a) \( \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2 + 5} \)
   - (b) \( \sum_{n=0}^{\infty} \frac{(-1)^{n+1} \sqrt{n}}{n + 4} \)
   - (c) \( \sum_{n=2}^{\infty} \frac{\cos(\pi n)}{\sqrt{n}} \)
   - (d) \( \sum_{n=1}^{\infty} (-1)^n \sin \left( \frac{\pi}{n} \right) \)
   - (e) \( \sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n} \)

4. Let \( S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \).
   - (a) \( S_{15} = 0.8245417573 \ldots \)
   - How close does the alternating series remainder theorem guarantee \( S_{15} \) to be from \( S \)?
   - (b) If you want the partial sum to be within 0.0001 of the sum, how many terms do you need to take?

5. Let \( S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} \).
   - (a) \( S_{15} = 0.90167618 \ldots \)
   - How close does the alternating series remainder theorem guarantee \( S_{15} \) to be from \( S \)?
   - (b) If you want the partial sum to be within 0.001 of the sum, how many terms do you need to take?