Math 132 - November 11, 2016

Section 11.6: Absolute Convergence

- Definition: $\sum a_n$ is **absolutely convergent** if $\sum |a_n|$ converges.
- Definition: $\sum a_n$ is **conditionally convergent** if $\sum a_n$ converges but $\sum |a_n|$ diverges.
- Theorem: If $\sum a_n$ is absolytely convergent then the series is convergent.

Section 11.5: Alternating Series

Let $S = \sum(-1)^n b_n$ where $b_n > 0$ be an alternating series. S converges if the following are true:

- $b_{n+1} < b_n$ (so the b_n are decreasing)
- $\lim b_n = 0$

And $|R_n| = |S - S_n| \leq b_{n+1}$

Warm-up Problems

1. $\boxed{\textbf{Clicker}}$ Does the series converge absolutely: $\displaystyle\sum_{i=1}^{\infty} \frac{(-1)^{n+1}}{n^2+2n+1}$

   (a) Yes (b) No (c) It converges conditionally (d) IDK but I'm moving to Canada with Amy Schumer

Class Problems

2. **Alternating Series:** Does the series converge or diverge:
   $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \cdots$

3. Are the following absolutely convergenct? Are they conditionally convergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+5}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-3}\sqrt{n}}{n+4}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{\sqrt{n}}$

   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sin(\pi/n)$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$

4. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$.

   (a) $S_{15} = 0.8245417573\ldots$.
   How close does the alternating series remainder theorem guarantee S_{15} to be from S?

   (b) If you want the partial sum to be within 0.0001 of the sum, how many terms do you need to take?

5. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$.

   (a) $S_{15} = 0.901676118\ldots$.
   How close does the alternating series remainder theorem guarantee S_{15} to be from S?

   (b) If you want the partial sum to be within 0.001 of the sum, how many terms do you need to take?