Math 132 Discussion Session: Week 11

1. Determine whether the following series converge or diverge. State any tests used and show that all conditions of the test are satisfied.

   a. $\sum_{n=1}^{\infty} \frac{1}{(n^2+2)^{\frac{3}{5}}}$ converges

   b. $\sum_{n=2}^{\infty} \frac{3^n+n}{2^n+4}$ diverges

11.3: The Integral Test and 11.4: Comparison Tests

2. Determine whether the following series converge or diverge using any of the methods discussed so far in class.

   a. $\sum_{n=1}^{\infty} \frac{\sin^4(n+1)}{n^2+1}$ converges

   b. $\sum_{n=1}^{\infty} \frac{1}{3^{n^2}}$ converges

   c. $\sum_{n=1}^{\infty} \frac{6^{n-3}}{5^n-n}$ diverges

   d. $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n^{\frac{3}{2}}}$ converges

   e. $\sum_{n=1}^{\infty} 3^{\ln n}$ diverges

   f. $\sum_{n=1}^{\infty} ne^{n^2}$ diverges

   g. $\sum_{n=2}^{\infty} \frac{1}{n^n}$ converges

3. For which values of $a > 0$ does $\sum_{n=2}^{\infty} \frac{1}{n^a \ln n}$ converge? $a > 1$

4. For which values of $a > 0$ does $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^a}$ converge? $a > 1$

5. For which values of p does $\sum_{n=2}^{\infty} \frac{n^2}{(n^3+1)^p}$ converge? $p > 1$

Solutions

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Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and write down your answer to #1 and include your work. Turn **this paper** in at the end of class. You do not need to turn in the question page or answers to the other questions.

Additional Instructions: It is okay if you do not completely finish all of the problems, but you should solve most of the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

Group Members

Group Answer and Work

1. Determine whether the following series converge or diverge. State any tests used and show that all conditions of the test are satisfied.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n^2+2)^{\frac{3}{5}}}$

Note that $\dfrac{1}{(n^2+2)^{3/5}} \leq \dfrac{1}{n^{6/5}}$ & $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{6/5}}$ converges b/c it's a p-series w/ $p = 6/5 > 1$.

By the Comparison Test, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n^2+2)^{5/5}}$ also converges

b. $\displaystyle\sum_{n=2}^{\infty} \frac{3^n+n}{2^n+4}$

$$\lim_{n\to\infty} \frac{3^n+n}{2^n+4} \cdot \frac{\frac{1}{3^n}}{1/3^n} = \lim_{n\to\infty} \frac{1+\frac{n}{3^n}}{[2/3]^n + 4/3^n} = \infty$$

$1/0$

Thus, $\displaystyle\sum_{n=2}^{\infty} \frac{3^n+n}{2^n+4}$ diverges by the n^{th}-term test