Answer key

Math 132: Discussion Session: Week 10

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and write down your answer to #1*a* and include your work. Turn **this paper** in at the end of class. You do not need to turn in the question page or answers to there other questions.

Additional Instructions: It is okay if you do not completely finish all of the problems, but you should solve most of the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

Group Members

Group Answer and Work

1. a. Compute the limit of the sequence $a_n = \dfrac{(\ln n)^2}{\sqrt{n}}$.

$$\lim_{n\to\infty} \frac{(\ln n)^2}{\sqrt{n}} = \lim_{x\to\infty} \frac{(\ln x)^2}{\sqrt{x}} \overset{\infty/\infty}{=} \overset{L'H}{\lim_{x\to\infty}} \frac{2\cdot \ln x \cdot 1/x}{1/2\, x^{-1/2}}$$

$$= \lim_{x\to\infty} \frac{4\ln x}{\sqrt{x}} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{4/x}{1/2\, x^{-1/2}} = \lim_{x\to\infty} \frac{8}{\sqrt{x}} = 0$$

b. If possible, determine the value of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2\cdot 3^n}{(-4)^n}$.

$$\sum_{n=1}^{\infty} 2\cdot\left[-3/4\right]^n = \frac{2\cdot[-3/4]}{1--3/4} = \frac{-6/4}{7/4} = -6/7$$

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1. a. Compute the limit of the sequence $a_n = \dfrac{(\ln n)^2}{\sqrt{n}}$. $\lim_{n\to\infty} a_n = 0$

   b. If possible, determine the value of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2\cdot 3^n}{(-4)^n}$. $= -6/7$

11.1: Sequences

2. Determine whether the sequence converges or diverges. If it converges, find the limit.

   a. $a_n = \tan^{-1}(\ln n)$ $\lim_{n\to\infty} a_n = \pi/2$

   b. $a_n = \ln(n+1) - \ln(n)$ $\lim_{n\to\infty} a_n = 0$

   c. $a_n = \left(1 + \frac{2}{n}\right)^n$ $\lim_{n\to\infty} a_n = e^2$

   d. $a_n = n - \sqrt{n+1}\sqrt{n+3}$ $\lim_{n\to\infty} a_n = -2$

   e. $a_n = n\sin\left(\frac{3}{n}\right)$ $\lim_{n\to\infty} a_n = 3$

11.2: Series

1. Determine whether the series converges or diverges. If it converges, determine its value.

   a. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$ Diverges by nth term test

   b. $\displaystyle\sum_{n=1}^{\infty} \frac{6\cdot 2^{2n-1}}{3^n} = \sum_{n=1}^{\infty} 6/2 \cdot \frac{4^n}{3^n}$ diverges b/c $r > 1$.

   c. $\displaystyle\sum_{n=1}^{\infty} (\sin 100)^n = \frac{\sin(100)}{1-\sin(100)}$

   d. $\displaystyle\sum_{n=1}^{\infty} \frac{2+n}{1-2n}$ diverges by nth Term Test

2. Find the values of x for which the series converges. Determine the value of the series for those values of x.

   a. $\displaystyle\sum_{n=1}^{\infty} (-5)^n x^n = \frac{5x}{1+5x}$ for x with $|x| < \frac{1}{5}$ or $-1/5 < x < 1/5$

   b. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n} = \frac{1}{1-\left(\frac{x-2}{3}\right)}$ for x with $-1 < x < 5$

   $\left|\frac{x-2}{3}\right| < 1 \Rightarrow -1 < \frac{x-2}{3} < 1 \Rightarrow -3 < x-2 < 3$