Oct. 31**st:** 11.2 : **Series**

Warm-Up

1 Compute the following sums. Use a calculator to write your answer as a decimal.

a. $\displaystyle\sum_{n=1}^{2} \frac{1}{2^n}$

b. $\displaystyle\sum_{n=1}^{6} \frac{1}{2^n}$

c. $\displaystyle\sum_{n=1}^{12} \frac{1}{2^n}$

In-Class Exercises

Defn. The Nth *partial sum* of the series $\displaystyle\sum_{n=1}^{\infty} a_n$ is

$$S_N = a_1 + \cdots + a_N = \sum_{n=1}^{N} a_n.$$

If $\displaystyle\lim_{N\to\infty} \{S_N\} = L$, then we say the series $\displaystyle\sum_{n=1}^{\infty} a_n$ *converges* and equals L. If $\displaystyle\lim_{N\to\infty} \{S_N\}$, does not exist, we say the series $\displaystyle\sum_{n=1}^{\infty} a_n$ *diverges.*

1. Find the value of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}$ or show that it diverges.

2. (Clicker) Let a and r be fixed constants. For which values of r does the geometric series $\displaystyle\sum_{n=0}^{\infty} ar^n$ converge?

   a. all r
   b. $0 \le r \le 1$
   c. $-1 \le r \le 1$
   d. $-1 < r \le 1$
   e. $-1 < r < 1$

3. Determine whether the following series converge or diverge. If they converge, find their value.

a. $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-2)^n}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

c. $\displaystyle\sum_{n=0}^{\infty} \ln\left(\frac{n}{n+1}\right)$

d. $\displaystyle\sum_{n=0}^{\infty} \frac{(-\pi)^n}{e^n}$

Properties of Series. Re-indexing Rule: If $\{a_n\}$ is a sequence, then $\displaystyle\sum_{n=k}^{\infty} a_n = \sum_{n=0}^{\infty} a_{n+k}$

4. Let $-1 < r < 1$ and let k be a positive integer. Find a formula for $\displaystyle\sum_{n=k}^{\infty} ar^n$.