Oct. 31st: 11.2 : Series

Warm-Up

1 Compute the following sums. Use a calculator to write your answer as a decimal.

   a. \( \sum_{n=1}^{2} \frac{1}{2^n} \)

   b. \( \sum_{n=1}^{6} \frac{1}{2^n} \)

   c. \( \sum_{n=1}^{12} \frac{1}{2^n} \)

In-Class Exercises

Defn. The \( N^{th} \) partial sum of the series \( \sum_{n=1}^{\infty} a_n \) is

\[
S_N = a_1 + \cdots + a_N = \sum_{n=1}^{N} a_n.
\]

If \( \lim_{N \to \infty} \{S_N\} = L \), then we say the series \( \sum_{n=1}^{\infty} a_n \) converges and equals \( L \). If \( \lim_{N \to \infty} \{S_N\} \), does not exist, we say the series \( \sum_{n=1}^{\infty} a_n \) diverges.

1. Find the value of the series \( \sum_{n=1}^{\infty} \frac{1}{2^n} \) or show that it diverges.

2. (Clicker) Let \( a \) and \( r \) be fixed constants. For which values of \( r \) does the geometric series \( \sum_{n=0}^{\infty} ar^n \) converge?
   
   a. all \( r \)
   
   b. \( 0 \leq r \leq 1 \)
   
   c. \( -1 \leq r \leq 1 \)
   
   d. \( -1 < r < 1 \)
   
   e. \( -1 < r < 1 \)
3. Determine whether the following series converge or diverge. If they converge, find their value.

a. \[ \sum_{n=0}^{\infty} \frac{3^{n+1}}{(-2)^n} \]

b. \[ \sum_{n=1}^{\infty} \frac{3}{n(n+3)} \]

c. \[ \sum_{n=0}^{\infty} \ln \left( \frac{n}{n+1} \right) \]

d. \[ \sum_{n=0}^{\infty} \frac{(-\pi)^n}{e^n} \]

Properties of Series. Re-indexing Rule: If \( \{a_n\} \) is a sequence, then \( \sum_{n=k}^{\infty} a_n = \sum_{n=0}^{\infty} a_{n+k} \)

4. Let \(-1 < r < 1\) and let \( k \) be a positive integer. Find a formula for \( \sum_{n=k}^{\infty} ar^n \).