Math 132 - October 28, 2016

Sections: 11.1

Terms of the Day

- Increasing sequence
- Decreasing Sequence
- Monotonic Sequence
- Bounded Above Sequence
- Bounded Below Sequence
- Bounded Sequence

Warm-up Problems

1. $\boxed{\textbf{Clicker}}$ Find $\lim_{n\to\infty} \frac{5!(n+2)!}{(n+3)!}$

   (a) 0 (b) $\frac{2}{3}$ (c) ∞ (d) $-\infty$

2. (From last time) Let $a_n = x^n$. Find $\lim_{n\to\infty} a_n$.

3. Find the derivative $\frac{d}{dn}\left(\frac{n+1}{n-1}\right) =$

4. ($e - \ln$ trick practice) Find $\lim_{n\to\infty} \left(\frac{n+1}{n-1}\right)^n$

5. Fun problem 1 (Continued Fractions). Find the limit of the sequence:

   $a_1 = 1$, $a_2 = 1 + \frac{1}{1}$, $a_3 = 1 + \cfrac{1}{1+\cfrac{1}{1}}$, $a_4 = 1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}$, $a_5 = 1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}$, etc.

6. Fun problem 2 (Continued Fractions). Find the limit of the sequence:

   $a_1 = 2$, $a_2 = 2 + \frac{1}{1}$, $a_3 = 2 + \cfrac{1}{1+\cfrac{1}{2}}$, $a_4 = 2 + \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1}}}$, $a_5 = 2 + \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1}}}}$, $a_6 = 2 + \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}}$,

   $a_7 = 2 + \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1}}}}}}$, $a_8 = 2 + \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1}}}}}}}$, etc.

   In this sequence, the "diagonal" of the fractions follows the pattern: $[2; 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, \ldots]$

Class Problems

7. For each of the sequences, find $\lim_{n\to\infty} a_n$

   (a) Let x be fixed and let $a_n = \dfrac{x^n}{n!}$.

   (b) $a_n = \dfrac{n!}{n^n}$

8. For each of the following sequences, determine if it is bounded (above/below/both), monotonic/increasing/decreasing.

   (a) $a_n = \dfrac{1}{n}$

   (b) $a_n = \dfrac{-1}{n}$

   (c) $a_n = \dfrac{(-1)^n}{n}$

   (d) $a_n = \dfrac{n+1}{n}$

   (e) $a_n = \dfrac{n+1}{n+2}$

   (f) $a_n = n^2$

   (g) $a_n = -n^2$

   (h) $a_n = (-1)^n n^2$

Monotonic Sequence Theorem: Every bounded monotonic sequence converges.

9. Determine if the following sequence is monotonic and bounded: $a_n = \frac{5n-2}{7n+3}$.

10. Experiment with the following recursive sequences. In particular, write down a few tersm, determine (if possible): monotonicity (increasing/decreasing), bounded (above/below), convergence (converge/diverge) and find an explicit formula for the terms.

    (a) $a_1 = 4$, $a_n = a_{n-1} + 5$

    (b) $a_1 = 16$, $a_n = \frac{1}{2}a_{n-1}$

    (c) (From the text) $a_1 = 1$, $a_n = \frac{1}{2}(a_{n-1} + 6)$

    (d) $a_1 = 0$, $a_2 = 1$, $a_{n+1} = a_n + 2a_{n-1}$

    (e) For the sequence from Probelm 10d, find $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$