Solutions

Oct. 26th: 11.1 : Sequences

Warm-Up

1 Find the next four terms of the following sequences:

a. 1, 3, 5, 7, 9 . . . 11, 13, 15, 17

b. $1, -4, 9, -16, 25, \ldots$ $-36, 49, -64, 81$

c. $\frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13} \cdots$ $\frac{7}{15}, \frac{8}{17}, \frac{9}{19}, \frac{10}{21}$

In-Class Exercises

Defn. A *sequence* is an infinite ordered list of numbers.

1. Find formulas for the n^{th} term of the following sequences:

a. 1, 3, 5, 7, 9 . . . $a_n = 2n-1$

b. $1, -4, 9, -16, 25, \ldots$ $b_n = (-1)^{n+1} n^2$

c. $\frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \ldots$ $c_n = \frac{n+1}{2n+3}$

Defn: Sequence Vocabulary. Let $\{a_n\}$ be a sequence.

i. $\{a_n\}$ is *bounded above* if there is some M such that $a_n \leq M$ for all n.

ii. $\{a_n\}$ is *bounded below* if there is some M such that $a_n \geq M$ for all n.

iii. $\{a_n\}$ is *bounded* if it is both bounded above and bounded below.

iv. $\{a_n\}$ is *increasing* if $a_{n+1} \geq a_n$ for all n.

v. $\{a_n\}$ is *decreasing* if $a_{n+1} \leq a_n$ for all n.

vi. $\{a_n\}$ is *monotonic* if it is either increasing or decreasing.

2. (Clicker) Consider the sequences given by the following formulas.

$$a_n = (-1)^n \frac{2n}{n+1}, \quad b_n = \frac{n^2}{\cancel{n!}\ (n+2)!} \quad c_n = \frac{\sin(\pi n)}{\ln(n+1)} \quad d_n = \frac{e^n}{n}.$$

Which of those sequences are bounded?

a. $\{a_n\}$, $\{c_n\}$, and $\{d_n\}$

$|a_n| = \frac{2n}{n+1} \leq \frac{2n}{n} = 2 \Rightarrow -2 \leq a_n \leq 2$

$0 \leq b_n = \frac{n \cdot n}{(n+2)(n+1) \cdot n \cdot \cdots 2 \cdot 1} \leq \frac{n}{n+2} \cdot \frac{n}{n+1} \leq 1$

$|c_n| \leq \frac{1}{\ln(n+1)} \leq \frac{1}{\ln 2} \Rightarrow -\frac{1}{\ln 2} \leq c_n \leq \frac{1}{\ln 2}$

b. $\{a_n\}$ and $\{b_n\}$

c. $\{b_n\}$ and $\{c_n\}$

d. $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$

e. All of them

3. (Clicker) Consider the sequences given by the following formulas.

$$a_n = (-1)^n \frac{2n}{n+1}, \quad b_n = \frac{n^2}{(n+2)!} \quad c_n = \frac{\sin(\pi n)}{\ln(n+1)} \quad d_n = \frac{e^n}{n}.$$

Which of those sequences are monotonic?

b_n = decreasing $\qquad$ d_n = increasing

a. $\{a_n\}$, $\{b_n\}$, and $\{d_n\}$

b. $\{a_n\}$ and $\{c_n\}$

c. $\{b_n\}$ and $\{d_n\}$

d. All of them

e. None of them

4. Determine whether the sequence converges or diverges. If it converges, find the limit.

a. $a_n = \frac{1}{n^2}$ $\qquad$ $\lim_{n\to\infty} \frac{1}{n^2} = 0$

b. $b_n = (-1)^n \frac{n+1}{n}$ $\qquad$ some $\to 1$, some $\to -1$ so diverges

5. Determine whether the sequence converges or diverges. If it converges, find the limit.

on Friday

a. $a_n = \frac{3n+1}{\sqrt{n+1}}$

b. $b_n = \frac{\ln(n)}{n}$

c. $c_n = \frac{\tan^{-1}(n)}{n+2}$

d. $d_n = e^{-2/\sqrt{n}}$

e. $e_n = \left(1 + \frac{2}{n}\right)^n$

f. $f_n = \frac{4^n}{1+9^n}$