Exam 2 Practice: Compilation of Problems

Below is a list of additional practice problems. Many of these are problems from our in-class worksheets that I did not have time to work through during class. You can find the solutions to these worksheets in Blackboard. Several questions are from discussion section worksheets - the answers to those questions are posted on the course syllabus. A couple are from the recommended problems in the textbook.

6.3: The Shell Method

Using the shell method, compute the volume of the solid of the region obtained by rotating the region enclosed about the given curves about the given vertical line.

a. $y = (x^2+1)^{-2}$, $y = 2-(x^2+1)^{-2}$, $x = 2$, about the y-axis (Discussion Worksheet Week 5)

b. $y = a - x$, with $a > 0$, $x = 0$, $x = a$, about $x = -1$ (Discussion Worksheet Week 5)

c. $y = 1 - x^2$, $x = -1$, $x = 1$, about $x = -1$ (9-21 Worksheet)

d. $x = \sqrt{y}$, $x = -y$ and $y = 2$, about $y = 6$ (9-21 Worksheet)

6.5: Average Value

a. Compute the average value of $g(x) = e^{-nx}$ on $[-1, 1]$. (Discussion Worksheet Week 5)

b. Assume $f(x) \leq 0$ and the area between $y = f(x)$ and the axis from $x = 2$ to $x = 9$ is 25. What is the average value of $f(x)$ on $[2, 9]$?

7.1: Integration By Parts

Evaluate the following integrals:

a. $\displaystyle\int x \tan^2 x \, dx$ (pp. 477 # 20)

b. $\displaystyle\int_0^{\pi/3} \sin x \ln(\cos x) \, dx$ (pp. 477 # 33)

c. $\displaystyle\int e^{\sqrt{x}} \, dx$ (Discussion Worksheet Week 5)

7.2: Trigonometric Integrals

Evaluate the following integrals:

a. $\displaystyle\int \sec^4(x) \, dx$ (9-26 worksheet)

b. $\displaystyle\int \csc^3(x) \, dx$ (Discussion Worksheet Week 6)

c. $\displaystyle\int_0^{\pi} \cos^4(2t) \, dt$ (pp. 484 # 9)

7.3: Trigonometric Substitution

Evaluate the following integrals:

a. $\displaystyle\int \frac{dx}{(x^2+a^2)^2}$ for $a > 0$ (Discussion Worksheet Week 6)

b. $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}}$ (9-30 Worksheet)

c. $\displaystyle\int \frac{dx}{\sqrt{25x^2-4}}$ (10-3 Worksheet)

7.4: The Partial Fraction Method

Evaluate the following integrals:

a. $\displaystyle\int \frac{x^2}{x^2-9}\,dx$ (10-5 Worksheet)

b. $\displaystyle\int \frac{dx}{(x+1)(x^2+2x+3)}$ (10-7 Worksheet)

c. $\displaystyle\int \frac{x^2+x-2}{3x^3-x^2+3x-1}\,dx$ (10-7 Worksheet)

7.5: Integration Strategies

Evaluate the following integrals:

a. $\displaystyle\int \sqrt{3-2x-x^2}\,dx$ (10-10 Worksheet)

b. $\displaystyle\int \frac{\cos(\frac{1}{x})}{x^3}\,dx$ (10-10 Worksheet)

c. $\displaystyle\int \frac{1+\tan x}{\sec x-1}\,dx$ (10-10 Worksheet)

7.8: Improper Integrals

Determine if the following integrals converge or diverge. If they converge, determine their value.

a. $\displaystyle\int_{-\infty}^{\infty} \frac{1}{9+x^2}\,dx$ (10-12 Worksheet)

b. $\displaystyle\int_{1}^{3} \frac{1}{(x-2)^2}\,dx$ (10-12 Worksheet)

c. $\displaystyle\int_{-1}^{0} \frac{e^{\frac{1}{x}}}{x^3}\,dx$ (pp. 535 # 39)

c. Use the Comparison Test to determine whether the integrals

$$\int_1^\infty \frac{1+\sin^2(x)}{\sqrt{x}}\,dx \quad \text{and} \quad \int_0^\infty \frac{\arctan(x)}{2+e^x}\,dx$$

each converge or diverge. (pp. 535 #'s 50, 52)

8.1: Arclength

Compute the length of the following curves:

a. $y = \frac{1}{2}x^2$ from $x = 0$ to $x = 1$ (10-19 Worksheet)

b. $y = \frac{x^4}{16} + \frac{1}{2x^2}$ for $1 \le x \le 2$ (10-19 Worksheet)

c. $y = 2\ln\left(\sin\left(\frac{x}{2}\right)\right)$ for $\frac{\pi}{3} \le x \le \pi$ (10-19 Worksheet)

8.2: Surface Area

a. Find the exact area of the surface obtained by rotating $y = \cos\left(\frac{x}{2}\right)$, $0 \le x \le \pi$ about the x-axis. (10-21 Worksheet)

b. Find the exact area of the surface obtained by rotating $x = e^{-y}$, $0 \le y \le 1$ about the y-axis. (10-21 Worksheet)

c. Find the exact area of the surface obtained by rotating $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $1 \le x \le 2$ about the y-axis. (10-21 Worksheet)