Answer Key

Oct. 12th: L'Hopital's Rule Practice

L'Hopital's Rule: Assume f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). If we have both

$$\lim_{x\to a} f(x) = 0 \text{ and } \lim_{x\to a} g(x) = 0 \quad \underline{\text{or}} \quad \lim_{x\to a} f(x) = \pm\infty \text{ and } \lim_{x\to a} g(x) = \pm\infty,$$

then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

Directions: Compute the following limits using L'Hopital's Rule.

1. $\lim\limits_{x\to 1} \frac{\ln x}{x-1}$ $0/0$ $= \lim\limits_{x\to 1} \frac{1/x}{1} = 1$

2. $\lim\limits_{x\to 0} \frac{\tan x - x}{x^3}$ $0/0$ $= \lim\limits_{x\to 0} \frac{\sec^2 x - 1}{3x^2}$ $0/0$ $= \lim\limits_{x\to 0} \frac{2\cdot\sec x\cdot\sec x\tan x}{6x}$ $0/0$

   $= \lim\limits_{x\to 0} \frac{2\cdot\sec x\cdot\sec x\tan x\cdot\tan x + 2\sec^4 x}{6} = 1/3$

3. $\lim\limits_{x\to +\infty} \sqrt{x}e^{-x/2}$ $+\infty\cdot 0$ $= \lim\limits_{x\to +\infty} \frac{\sqrt{x}}{e^{x/2}}$ ∞/∞ $= \lim\limits_{x\to\infty} \frac{1/2 x^{-1/2}}{1/2 e^{x/2}} = \lim\limits_{x\to\infty} \frac{1}{\sqrt{x}e^{x/2}} = 0$

4. $\lim\limits_{x\to\infty} \frac{\ln x}{x}$ ∞/∞ $= \lim\limits_{x\to\infty} \frac{1/x}{1} = \lim\limits_{x\to\infty} \frac{1}{x} = 0$

5. $\lim\limits_{x\to\infty} x^{3/2}\sin\left(\frac{1}{x}\right)$ $\infty\cdot 0$ $= \lim\limits_{x\to\infty} \frac{\sin(\frac{1}{x})}{x^{-3/2}} = \lim\limits_{x\to\infty} \frac{-1/x^2\cos(1/x)}{-3/2\, x^{-5/2}}$

   $= \lim\limits_{x\to\infty} \frac{2}{3}\cdot\frac{x^{5/2}}{x^2}\cos(1/x) = \lim\limits_{x\to\infty} \frac{2}{3}\sqrt{x}\underbrace{\cos(\tfrac{1}{x})}_{\downarrow 1} = \infty$

6. $\lim\limits_{x\to 1^+} \ln x \tan\left(\frac{\pi x}{2}\right)$ $0\cdot\infty$ $= \lim\limits_{x\to 1^+} \frac{\ln x}{\cot(\pi x/2)}$ $0/0$ $= \lim\limits_{x\to 1^+} \frac{1/x}{-\pi/2\csc^2(\pi x/2)}$

   $= \lim\limits_{x\to 1^+} -\frac{2}{\pi}\cdot\frac{\sin^2(\pi x/2)}{x} = -\frac{2}{\pi}$