Math 132: Discussion Session: Week 7

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and write down your answer to #1 and include your work. Turn **this paper** in at the end of class. You do not need to turn in the question page or answers to there other questions.

Additional Instructions: If you do not finish all of the problems, you should work through the rest of them for practice. Each group member should work through each problem, as similar problems may appear on the exam. To receive credit for the day's assignment, you **cannot** leave discussion section early unless you have completely finished all of the worksheet problems.

Group Members

Group Answer and Work

7.4: The Method of Partial Fractions

1. a. (2 pts) Find the partial fraction decomposition of $\dfrac{x^5}{x^4 - 1}$.

$$x^4-1 \overline{\big)\ x^5+0x^4+0x^3+0x^2+0x+0} \quad \overset{X}{} \overset{RX}{}$$
$$-(\ x^5 - \qquad\qquad x)$$
$$\overline{ X}$$

$$\Rightarrow \frac{x^5}{x^4-1} = x + \frac{x}{x^4-1} = x + \frac{A}{x-1} + \frac{B}{x+1} + \frac{C+Dx}{x^2+1}$$

$$x^4-1 = (x^2-1)(x^2+1) = (x-1)(x+1)(x^2+1) \Rightarrow x = A(x+1)(x^2+1) + B(x-1)(x^2+1)$$
$$+ (C+Dx)(x+1)(x-1)$$

$$x=1 \Rightarrow 1 = A \cdot 4 \Rightarrow A = \tfrac{1}{4} \qquad x^0 \quad 0 = A - B - C \Rightarrow C = 0$$
$$x=-1 \Rightarrow -1 = B(-4) \quad B = \tfrac{1}{4} \qquad x^3 \quad 0 = A + B + D \Rightarrow D = -\tfrac{1}{2}$$

$$\frac{x^5}{x^4-1} = x + \frac{\tfrac{1}{4}}{x-1} + \frac{\tfrac{1}{4}}{x+1} - \frac{\tfrac{1}{2}x}{x^2+1}$$

b. (1 pt) Evaluate the integral $\displaystyle\int \frac{x^5}{x^4 - 1}\, dx$.

$$\int \frac{x^5}{x^4-1}\, dx = \int x + \frac{\tfrac{1}{4}}{x-1} + \frac{\tfrac{1}{4}}{x+1} - \frac{\tfrac{1}{2}x}{x^2+1}\, dx$$

$$= \frac{x^2}{2} + \tfrac{1}{4}\ln|x-1| + \tfrac{1}{4}\ln|x+1| - \tfrac{1}{4}\ln|x^2+1| + C$$

Math 132 Discussion Session: Week 7

7.4: The Method of Partial Fractions

1. a. Find the partial fraction decomposition of $\dfrac{x^5}{x^4 - 1}$.

   b. Evaluate the integral $\displaystyle\int \frac{x^5}{x^4 - 1}\, dx$.

2. Compute the following integrals using the method of partial fractions:

   a. $\displaystyle\int \frac{10}{(x-1)^2(x^2+9)}\, dx$ $\quad -\frac{1}{5}\ln|x-1| - \frac{1}{x-1} + \frac{1}{10}\ln|x^2+9| - \frac{4}{15}\arctan\left(\frac{x}{3}\right) + C$

   b. $\displaystyle\int \frac{x^3+1}{x^2+1}\, dx = \frac{x^2}{2} - \frac{1}{2}\ln|x^2+1| + \arctan(x) + C$

   c. $\displaystyle\int \frac{1}{(x+2)(x^2+4x+10)}\, dx = \frac{1}{6}\ln|x+2| - \frac{1}{12}\ln|x^2+4x+10| + C$

   d. $\displaystyle\int \frac{4x^2-20}{(2x+5)^3}\, dx = \frac{5}{2x+5} - \frac{5}{4(2x+5)^2} + \frac{1}{2}\ln|2x+5| + C$

7.5: Strategy for Integration

1. Compute the following integrals using the integration method of your choice:

   a. $\displaystyle\int x^3(\ln x)^2\, dx = \frac{1}{32}x^4 + \frac{1}{4}x^4(\ln x)^2 - \frac{1}{8}x^4\ln x + C$

   b. $\displaystyle\int (3\sec x - \cos x)^2\, dx = \frac{1}{4}\sin(2x) + 9\tan x - \frac{11}{2}x + C$

   c. $\displaystyle\int \frac{1+\sin x}{1-\sin x}\, dx = 2\sec x + 2\tan x - x + C$

   d. $\displaystyle\int \frac{x^2\, dx}{(x^2-1)^{3/2}} = \ln\left|\sqrt{x^2-1}+x\right| - \frac{x}{\sqrt{x^2-1}} + C$

   e. $\displaystyle\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\, dx. \quad \frac{2}{3}\left[(x+1)^{3/2} - x^{3/2}\right] + C$

   f. $\displaystyle\int \frac{xe^x}{\sqrt{1+e^x}}\, dx \quad 2(x-2)\sqrt{1+e^x} + 2\ln\left[\frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}\right] + C$