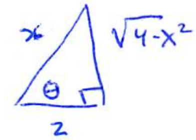
Answer Key

Oct. 3rd: 7.3 Trig Sub & 7.4 Partial Fractions

Finishing Trig Substitution

1. Compute the following integrals using trig substitution:

a. $\displaystyle\int \frac{dx}{x^3\sqrt{x^2-4}}$ $\quad X = 2\sec\theta \quad dx = 2\sec\theta\tan\theta\, d\theta \quad X^2-4 = 4\tan^2\theta$



$\sin(2\theta) = 2\cos\theta\sin\theta = 2\cdot\frac{2}{x}\cdot\frac{\sqrt{4-x^2}}{x}$

$$= \int \frac{2\sec\theta\tan\theta\, d\theta}{8\sec^2\theta\cdot 2\tan\theta} = \frac{1}{8}\int \cos^2\theta\, d\theta = \frac{1}{16}\int 1+\cos(2\theta)\, d\theta$$

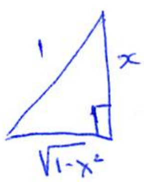
$$= \frac{1}{16}\theta + \frac{1}{32}\sin(2\theta) + C = \frac{1}{16}\operatorname{arcsec}\left(\frac{x}{2}\right) + \frac{2}{16}\frac{\sqrt{4-x^2}}{x} + C$$

b. $\displaystyle\int \frac{dx}{\sqrt{25x^2-4}} = \frac{1}{5}\int\frac{dx}{\sqrt{x^2-4/25}}$ $\quad X = 2/5\sec\theta \quad dx = 2/5\sec\theta\tan\theta\, d\theta \quad X^2-4/25 = \frac{4}{25}\tan^2\theta$

$$= \frac{1}{5}\int\frac{2/5\sec\theta\tan\theta\, d\theta}{2/5\tan\theta} = \frac{1}{5}\int\sec\theta\, d\theta = \frac{1}{5}\ln|\sec\theta+\tan\theta| + C$$

$$= 1/5 \ln\left|5x/2 + 5/2\sqrt{x^2-4/25}\right| + C$$

c. $\displaystyle\int \frac{x^3}{(1-x^2)^2}\, dx.$ $\quad X = \sin\theta \quad dx = \cos\theta\, d\theta \quad 1-x^2 = \cos^2\theta$



$$= \int\frac{\sin^3\theta\cos\theta\, d\theta}{\cos^4\theta} = \int\frac{\sin^3\theta}{\cos^3\theta}\, d\theta = \int\tan^3\theta\, d\theta = \int\tan\theta\left[\sec^2\theta-1\right] d\theta$$

$$= \frac{\tan^2\theta}{2} + \ln|\cos\theta| + C = \frac{x^2}{2(1-x^2)} + \ln\left|\sqrt{1-x^2}\right| + C$$

2. (Clicker) Do you want some time in class to practice trig sub yourself?

a. I'm good! Let's move on to the next topic.

b. I'd like five minutes to try one extra problem.

c. I'd like ten minutes to work an entire extra problem.

d. I'd like twenty minutes, but I know that's probably not going to happen.

Partial Fractions: Warm-Up

1. Combine the following rational functions into one rational function by getting a common denominator:

a. $\displaystyle\frac{2}{x+4} + \frac{3}{x+1} = \frac{2(x+1)+3(x+4)}{(x+4)(x+1)} = \frac{5x+14}{(x+4)(x+1)}$

b. $\displaystyle\frac{-1}{x} + \frac{1/2}{x-1} + \frac{1/2}{x+1} = \frac{-1(x-1)(x+1) + 1/2(x)(x+1) + 1/2(x)(x-1)}{x(x-1)(x+1)}$

$=$

Partial Fractions: In-Class Exercises

1. Using polynomial long division to write the following rational function in the form $Q(x)+\dfrac{p(x)}{q(x)}$, where $\deg p < \deg q$: $\dfrac{3x^4+3x^3-5x^2+x-1}{x^2+x-2}$.

$$3x^2+1 \quad R1$$

$$x^2+x-2 \,\big)\, 3x^4+3x^3-5x^2+x-1$$

$$-(3x^4+3x^3-6x^2)$$

$$x^2+x-1$$

$$-(x^2+x-2)$$

$$1$$

$$\boxed{3x^2+1+\frac{1}{x^2+x-2}}$$

2. (Clicker) Write $\frac{x^2+2}{x+3}$ in the form $Q(x)+\dfrac{p(x)}{q(x)}$, where $\deg p < \deg q$.

   a. $x+3+\dfrac{7}{x+3}$

   b. $7+\dfrac{x+3}{x-3}$

   c. $11+\dfrac{x-3}{x+3}$

   d. $x-3+\dfrac{11}{x+3}$

$$x-3 \quad R11$$

$$x+3 \,\big)\, x^2+0x+2$$

$$-(x^2+3x)$$

$$-3x+2$$

$$-(-3x-9)$$

$$11$$