Answer Key

Sept. ~~26~~ 28th: 7.2 Trigonometric Integrals

Warm Up Exercises

1. $\int \cos x \sin x \, dx = \int u \, du = u^2/2 + C = \frac{\sin^2 x}{2} + C$

   $u = \sin x$
   $du = \cos x \, dx$

2. $\int \tan^2 x \sec^2 x \, dx = \int u^2 du = \frac{u^3}{3} + C = \frac{\tan^3 x}{3} + C$

   $u = \tan x$
   $du = \sec^2 x \, dx$

Trigonometric Integrals

1. Type 1: Evaluate the following integrals:

a. $\int \sin^3(x) \cos^4(x) \, dx = \int \sin^2(x) \cos^4(x) \cdot \sin x \, dx = \int (1-\cos^2 x)\cos^4 x \cdot \sin x \, dx$

   $u = \cos x$
   $du = -\sin x \, dx$

   $= -\int (1-u^2)u^4 du = \int u^6 - u^4 du = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

b. $\int \cos^2(x) \, dx = \int \frac{1+\cos(2x)}{2} dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$

c. $\int \sin^6(x) \cos^5(x) \, dx = \int \sin^6(x) [1-\sin^2 x]^2 \cos x \, dx$

   $u = \sin x$
   $du = \cos x \, dx$

   $= \int u^6 (1 - 2u^2 + u^4) du = \int u^6 - 2u^8 + u^{10} du$

   $= \frac{\sin^7 x}{7} - \frac{2\sin^9 x}{9} + \frac{\sin^{11} x}{11} + C$

d. $\int \cos^2(x) \sin^2(x) \, dx$

   $= \int \left(\frac{1+\cos(2x)}{2}\right)\left(\frac{1-\cos(2x)}{2}\right) dx = \frac{1}{4}\int 1 - \cos^2(2x) \, dx$

   $= \frac{1}{4}x - \frac{1}{4}\int \frac{1+\cos(4x)}{2} dx = \frac{1}{4}x - \frac{1}{8}x - \frac{1}{32}\sin(4x) + C$

2. Type 2: Evaluate the following integrals:

a. $\int \tan^2(x)\sec^4(x)\,dx$ $= \int \tan^2 x\,[1+\tan^2 x]\sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x$

$= \int u^2(1+u^2)\,du = \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$

b. $\int \tan^5(x)\sec^3(x)\,dx$ $= \int (\sec^2 x - 1)^2 \sec^2 x \cdot \sec x \tan x\,dx$

$u = \sec x$

$du = \sec x \tan x\,dx$

$= \int (u^4 - 2u^2 + 1)u^2\,du = \int u^6 - 2u^4 + u^2\,du$

$= \frac{\sec^7 x}{7} - \frac{2\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$

c. $\int \sec(x)\,dx$ $= \int \sec(x) \cdot \frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{du}{u} = \ln|\sec x + \tan x| + C$

$u = \sec x + \tan x$

$du = \sec^2 x + \sec x \tan x\,dx$

d. $\int \sec^4(x)\,dx$ $= \int (1+\tan^2 x)\sec^2 x\,dx = \int \sec^2 x\,dx + \int u^2\,du$

$u = \tan x$

$du = \sec^2 x$

$= \tan x + \frac{1}{3}\tan^3 x + C$

3. Type 3: Evaluate the following integrals:

a. $\int \sin(3x)\cos(4x)\,dx$ $= \int \frac{1}{2}\sin(-x) + \frac{1}{2}\sin(7x)\,dx = \frac{1}{2}\cos x - \frac{1}{14}\cos(7x) + C$

b. $\int \sin(4x)\sin(-2x)\,dx$ $= -\int \sin(4x)\sin(2x)\,dx$

$= -\frac{1}{2}\int \cos(2x) - \cos(6x)\,dx$

$= -1/2\left[\frac{\sin(2x)}{2} - \frac{\sin(6x)}{6}\right] + C$

c. $\int \cos(5x)\cos(3x)\,dx$ $= \frac{1}{2}\int \cos(2x) + \cos(8x)\,dx$

$= 1/2\left[\frac{\sin(2x)}{2} + \frac{\sin(8x)}{8}\right] + C$