

Answer key

Sept. 26th: 7.1 By-Parts Integration

Warm Up Exercises

1. Evaluate the following integrals using integration by-parts:

a. $\int 3xe^{-x} dx$ $u=3x$ $dv=e^{-x}dx$ $integral = -3xe^{-x} - 3e^{-x} + C$
 $du=3$ $v=-e^{-x}$

b. $\int_0^{\pi/2} x^2 \sin(x) dx$ (Hint: Use by-Parts Integration twice.)
 $u=x^2$ $dv=\sin x dx$ (1st by parts) $integral = \pi - 2$
 $du=2x dx$ $v=-\cos x$

In-Class Exercises

1. (Clicker) Consider $\int \arcsin(x) dx$. If we are going to compute this integral via by-parts integration, what are good choices for u and dv ?

a. $u = 1$, $dv = \arcsin(x) dx$

b. $u = \arcsin(x)$, $dv = dx$

c. It doesn't seem like by-parts integration will work on this problem.

d. I vote for a corny math riddle.

2. Evaluate the following integrals using integration by-parts:

a. $\int \arcsin(x) dx$ $u=\arcsin(x)$ $dv=dx$ $integral = x \arcsin(x)$
 $du = \frac{1}{\sqrt{1-x^2}} dx$ $v=x$ $+ \sqrt{1-x^2} + C$

b. $\int \frac{\ln x}{x^2} dx$ $u=\ln x$ $dv=1/x^2 dx$ $integral = -\frac{\ln x}{x} - \frac{1}{x} + C$
 $du = \frac{1}{x} dx$ $v = -1/x$

c. $\int_1^{\sqrt{3}} \arctan(1/x) dx$ $u=\arctan(1/x)$ $dv=dx$ $integral = \frac{\sqrt{3}\pi}{6} - \frac{\pi}{4} - \frac{\ln 2}{2}$
 $du = (-\frac{1}{x^2}) \frac{1}{1+(1/x)^2} dx$ $v=x$
 $= \frac{-1}{x^2+1} dx$

d. $\int x \sin x \cos x dx$ $u=x$ $dv=\sin x \cos x dx$
 $du=dx$ $v=\frac{\sin^2 x}{2}$ $\text{integral} = \frac{x \sin^2(x)}{2} - \frac{1}{4}x$
 $+\frac{\sin(2x)}{8} + C$

e. $\int_0^1 (x^2+1)e^{-x} dx$ $u=(x^2+1)$ $dv=e^{-x} dx$
 $du=2x dx$ $v=-e^{-x}$ $\text{integral} = 3 - 6/e$
 (1st by parts)

f. $\int \cos(\ln(x)) dx$. (Hint: Use u-substitution and then integration by parts)

$u=\ln x$ $du=1/x dx$ $\int x \cos(u) du = \int e^u \cos(u) du$ $\text{integral} = \frac{x \cos(\ln x) + x \sin(\ln x)}{2}$
 (use #3 now)

3. (Clicker) Consider $\int e^x \cos x dx$. If we are going to compute this integral via by-parts integration, what are good choices for u and dv ? $+C$

either work

- a. $u = e^x, dv = \cos x dx$
 b. $u = \cos x, dv = e^x dx$

$$\int e^x \cos x dx = \frac{e^x(\cos x + \sin x)}{2} + C$$

c. $u = e^x \cos x, dv = dx$

d. $u = 1, dv = e^x \cos x dx$

Trigonometric Integrals

1. Evaluate the following integrals:

a. $\int \sin^3 \cos^4 x dx$

b. $\int \cos^2(x) dx$

c. $\int \sin^6 x \cos^5 x dx$

d. $\int \cos^2 x \sin^2(x) dx$

next class