Sept. 21st: 6.3: Shell Method

Warm Up Exercises

1. Graph the region between \( y = \sin x \) and \( y = 0 \) from \( x = 0 \) to \( x = \pi \)

2. Rotate the region from (1) about the \( y \)-axis to obtain a solid. Try to compute the volume of this solid using the disk or washer method. What goes wrong?

In-Class Exercises

1. (clicker) The region between \( y = \sin x \) and \( y = 0 \) from \( x = 0 \) to \( x = \pi \) is revolved about the \( y \)-axis to form a solid. For \( i = 1, \ldots, n \), what is the shell radius \( r(x_i) \) and shell height \( h(x_i) \) of the \( i \)th shell?
   
   a. \( r(x_i) = x_i \), \( h(x_i) = \sin^{-1}(x_i) \)
   
   b. \( r(x_i) = \frac{x_i}{2} \), \( h(x_i) = \sin^{-1}(x_i) \)
   
   c. \( r(x_i) = x_i \), \( h(x_i) = \sin(x_i) \)
   
   d. \( r(x_i) = \frac{x_i}{2} \), \( h(x_i) = \sin(x_i) \)

Shell Method. Assume a solid \( S \) is obtained by revolving a region between \( x = a \) and \( x = b \) around a vertical line. Then

\[
\text{Volume}(S) = \int_a^b 2\pi r(x)h(x) \, dx,
\]

where \( r(x) \) is the shell radius and \( h(x) \) is the shell height of the resulting cylindrical shell formed at \( x \).

2. (clicker) The region between \( x = y^2 \) and \( x = 4 \) is revolved about the line \( x = 5 \) to form a solid. Fix \( x \). What is the shell radius \( r(x) \) and shell height \( h(x) \) for the cylindrical shell formed at \( x \)?

   a. \( r(x) = x \), \( h(x) = x^2 \)
   
   b. \( r(x) = 5 - x \), \( h(x) = \sqrt{x} \)
   
   c. \( r(x) = 5 - x \), \( h(x) = 2\sqrt{x} \)
   
   d. \( r(x) = x \), \( h(x) = \sqrt{x} \)
3. Using the shell method, compute the volume of the solid of the region obtained by rotating the region enclosed about the given curves about the given vertical line.

   a. $y = x^3, y = 0, x = 0, x = 1,$ about $x = 2$
   
   b. $y = 3x - 2, y = 6 - x, x = 0$ about the $y$ axis
   
   c. $y = 1 - x^2, x = -1, x = 1,$ about $x = -1$

4. (clicker) The region between $x = \sqrt{y}, x = -y$ and $y = 2$ is is revolved about the line $x$-axis to form a solid. Fix $y$. What is the shell radius $r(y)$ and shell height $h(y)$ for the cylindrical shell formed at $y$?

   a. $r(y) = y, h(y) = \sqrt{y} + y$
   
   b. $r(y) = \sqrt{y}, h(y) = 2 - y$
   
   c. $r(y) = 2 - y, h(y) = \sqrt{y} - y$
   
   d. $r(y) = \sqrt{y} + y, h(y) = y$

5. Using the shell method, compute the volume of the solid of the region obtained by rotating the region enclosed about the given curves about the given horizontal line.

   a. $x = \sqrt{y}, x = -y$ and $y = 2,$ about the $x$-axis
   
   b. $x = \sqrt{y}, x = -y$ and $y = 2,$ about $y = 6$
   
   c. $y = 4 - x^2, x = 0, y = 0$ about the $x$-axis