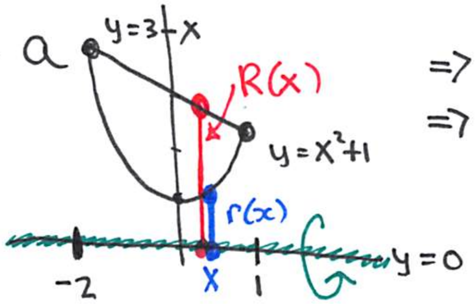
#2 Solutions 9-16 WS

#a



$$x^2+1=3-x$$
$$\Rightarrow x^2+x-2=0$$
$$\Rightarrow x=-2,1$$

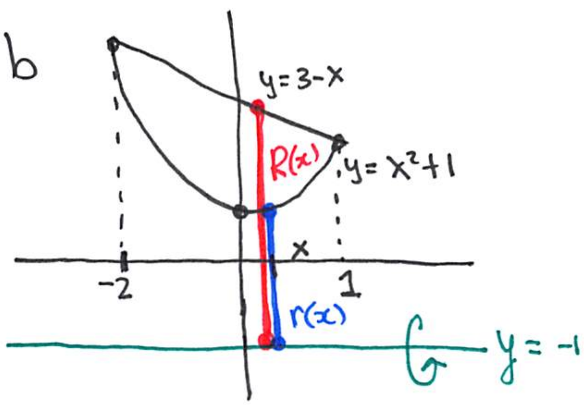
$-2 \le x \le 1$

$r(x) = x^2+1-0 = x^2+1$

$R(x) = 3-x-0 = 3-x$

$$\text{Vol} = \int_{-2}^{1} \pi(3-x)^2 - \pi(x^2+1)^2\,dx$$

#b



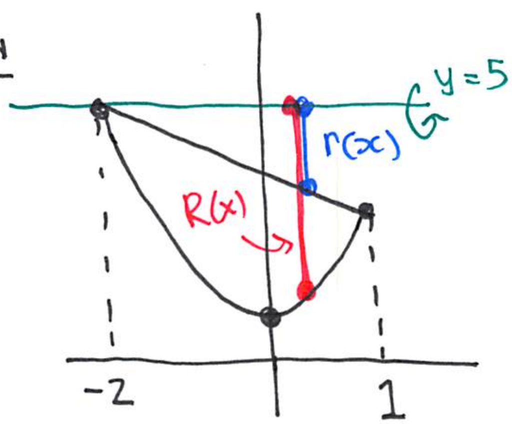
$-2 \le x \le 1$

$R(x) = (3-x)-(-1) = 4-x$

$r(x) = (x^2+1)-(-1) = x^2+2$

$$\text{Vol} = \int_{-2}^{1} \pi\left[(4-x)^2-(x^2+2)^2\right]dx$$

#c



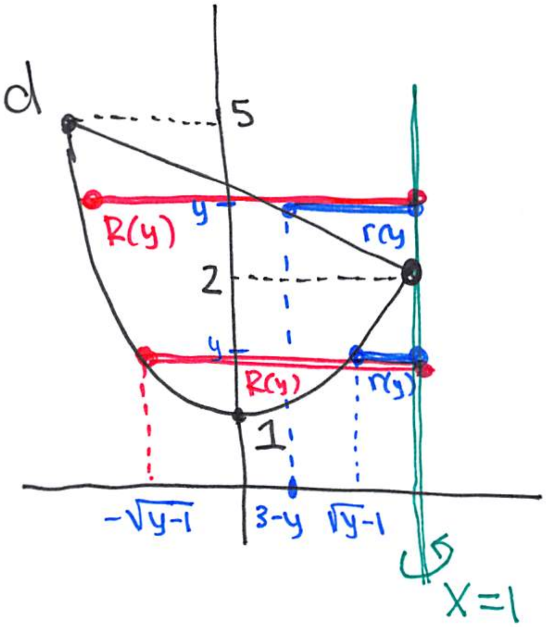
$-2 \le x \le 1$

$r(x) = 5-(3-x) = 2+x$

$R(x) = 5-(x^2+1) = 4-x^2$

$$\text{Vol} = \int_{-2}^{1} \pi\left[(4-x^2)^2-(x+2)^2\right]dx$$

d



For $1 \leq y \leq 2$

* $r(y) = 1 - \sqrt{y-1}$

* $R(y) = 1 - (-\sqrt{y-1}) = 1 + \sqrt{y-1}$

For $2 \leq y \leq 5$

* $r(y) = 1 - (3-y) = y-2$

* $R(y) = 1 - (-\sqrt{y-1}) = 1 + \sqrt{y-1}$

$y = x^2 + 1 \Rightarrow x = \pm\sqrt{y-1}$

$y = 3 - x \Rightarrow x = 3 - y$

$$\text{Vol} = \int_1^2 \pi \left[(1+\sqrt{y-1})^2 - (1-\sqrt{y-1})^2 \right] dy + \int_2^5 \pi \left[(1+\sqrt{y-1})^2 - (y-2)^2 \right] dy$$

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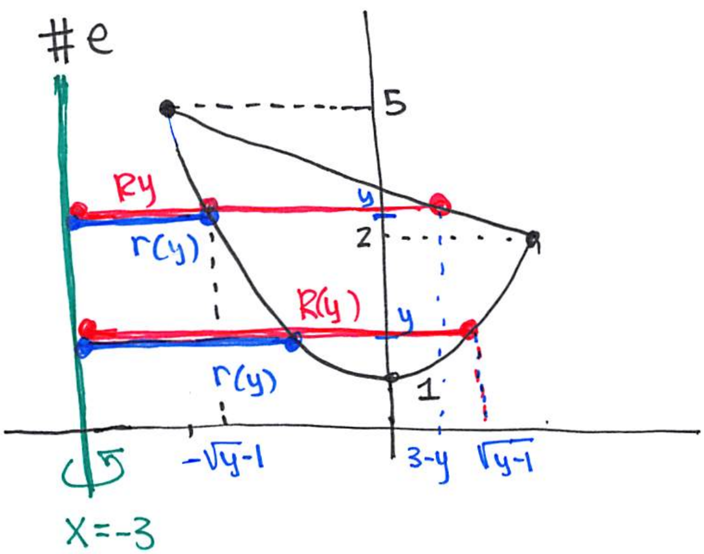
e



For $1 \leq y \leq 2$

* $r(y) = -\sqrt{y-1} - -3 = 3 - \sqrt{y-1}$

$R(y) = 3 - y - -3 = 6 - y$

For $2 \leq y \leq 5$

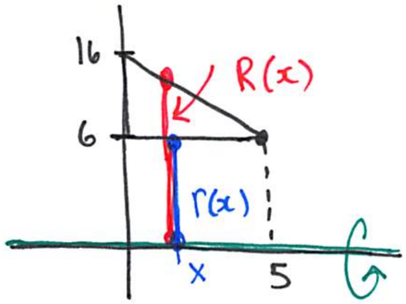
$r(y) = -\sqrt{y-1} - -3 = 3 - \sqrt{y-1}$

$R(y) = \sqrt{y-1} - -3 = 3 + \sqrt{y-1}$

$$\text{Vol} = \int_1^2 \pi \left[(6-y)^2 - (3-\sqrt{y-1})^2 \right] dy + \int_2^5 \pi \left[(3+\sqrt{y-1})^2 - (3-\sqrt{y-1})^2 \right] dy$$

#3 Answers

a) $y = 16 - 2x$, $y = 6$, $x = 0$ $\quad 16 - 2x = 6 \Rightarrow x = 5$



$0 \leq x \leq 5$

- $r(x) = 6 - 0 = 6$
- $R(x) = 16 - 2x - 0 = 16 - 2x$

$$\text{Vol} = \int_0^5 \pi \left[(16-2x)^2 - 36 \right] dx$$

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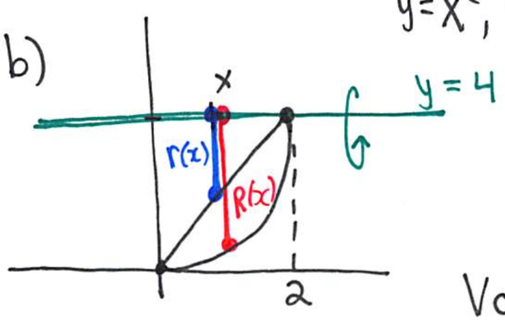
b) $y = x^2$, $y = 2x$ $\quad 2x = x^2 \Rightarrow x = 0, 2$



$0 \leq x \leq 2$

- $r(x) = 4 - 2x$
- $R(x) = 4 - x^2$

$$\text{Vol} = \int_0^2 \pi \left[(4-x^2)^2 - (4-2x)^2 \right] dx$$

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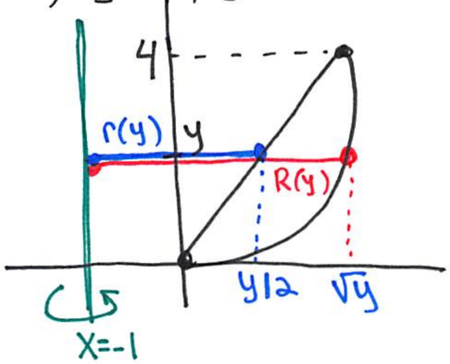
c) $y = x^2$, $y = 2x$ about $x = -1$ $\quad y = x^2 \Rightarrow x = \sqrt{y}$ $\quad y = 2x \Rightarrow x = y/2$

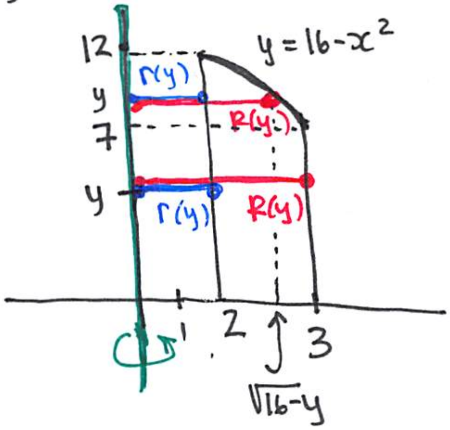


$0 \leq y \leq 4$

- $r(y) = y/2 - -1 = y/2 + 1$
- $R(y) = \sqrt{y} - -1 = \sqrt{y} + 1$

$$\text{Vol} = \int_0^4 \pi \left[(\sqrt{y}+1)^2 - (y/2+1)^2 \right] dy$$

d) $x=2,\ x=3,\ y=16-x^2 \Rightarrow x=\sqrt{16-y}$



$0 \leq y \leq 7$

- $r(y) = 2-0 = 2$
- $R(y) = 3-0 = 3$

$7 \leq y \leq 12$

- $r(y) = 2-0 = 2$
- $R(y) = \sqrt{16-y} - 0 = \sqrt{16-y}$

$$\text{Vol} = \int_0^7 \pi\left[9-4\right]dy + \int_7^{12} \pi\left[16-y-4\right]dy$$

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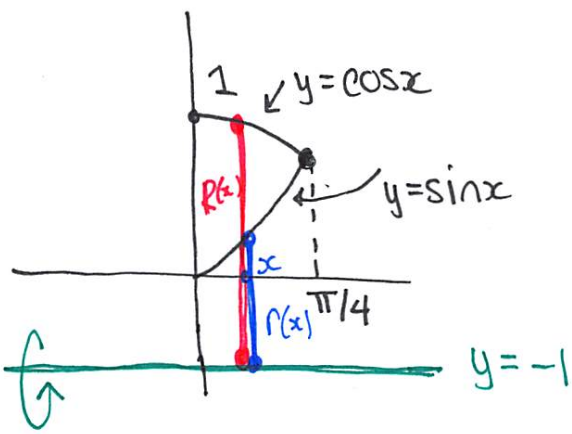
e) $y=\cos x,\ y=\sin x \quad 0 \leq x \leq \frac{\pi}{4}$



$0 \leq x \leq \frac{\pi}{4}$

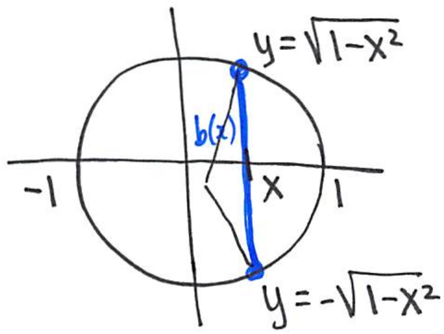
- $r(x) = \sin x - -1 = \sin x + 1$
- $R(x) = \cos x - -1 = \cos x + 1$

$$\text{Vol} = \int_0^{\pi/4} \pi\left(\left[\cos x + 1\right]^2 - \left[\sin x + 1\right]^2\right) dx$$

Additional Practice (computing volumes that are not solids of revolution)

#1

a)



Need $A(x)$ = cross sectional area

Given: cross sections are triangles with base = height

So $A(x) = \frac{(\text{base at } x)^2}{2}$

From the picture the base of the triangle at x, $b(x) = 2\sqrt{1-x^2}$

so $A(x) = \left(2\sqrt{1-x^2}\right)^2/2 = 2(1-x^2)$

and $\text{Vol} = \int_{-1}^{1} A(x)\,dx = \int_{-1}^{1} 2(1-x^2)\,dx = 2x - \frac{2x^3}{3}\Big|_{-1}^{1} = \frac{8}{3}$

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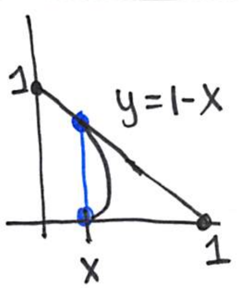
b)



Need $A(x)$ = cross sectional area

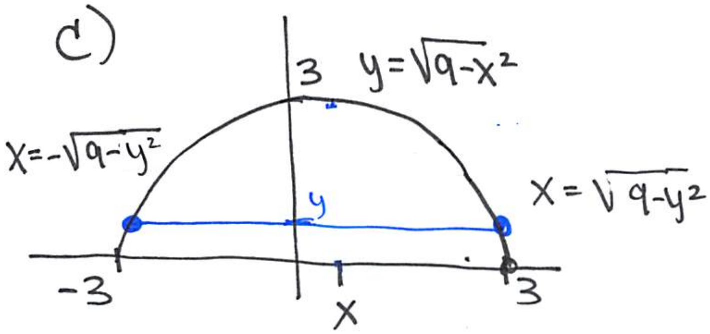
Given: cross sections are semicircles.

Fix x:

From the picture, the diameter of the semicircle at x is $1-x$ so the radius is $\frac{1-x}{2}$

$\Rightarrow \underbrace{A(x) = \pi\left(\frac{1-x}{2}\right)^2 \cdot \frac{1}{2}}_{\text{area of semicircle}} = \frac{\pi(1-x)^2}{8}$

and $\text{Vol} = \int_0^1 \frac{\pi(1-x)^2}{8}\,dx$ = pi/ 24 c)



Need: $A(y)$ = area of cross section

cross sections perpendicular to y axis are squares.

For a fixed y, the side length of the square is:

$$\sqrt{9-y^2} - -\sqrt{9-y^2} = 2\sqrt{9-y^2}$$

So area $A(y) = 4(9-y^2)$

and $\quad \text{Vol} = \int_0^3 4(9-y^2)\,dy = 72$