Sept. 12th: 6.1 Area and 6.2 Volume

Warm Up Exercises

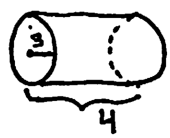
1. Write the following areas using integrals. Time permitting, compute the areas.

   a. The area of the region between $f(x) = \frac{1}{x}$ and $y = 0$ (the x-axis) from $x = 2$ to $x = 3$.

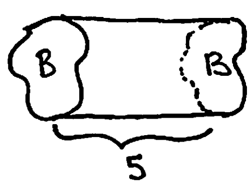
   b. The area of the region bounded by the curves $y = x - 2$, the x-axis, $x = 1$, and $x = 3$.

2. Compute the volumes of the following solids:

   a. Solid 1:



   b. Solid 2:



area(B) = $3/2\pi$

In-Class Exercises

Area Rule. If $f(x)$ and $g(x)$ are continuous and if $f(x) \geq g(x)$ for all x in $[a, b]$, then the area of the region R bounded by the curves $y = f(x)$ and $y = g(x)$, $x = a$, and $x = b$ is

$$\text{Area of } R = \int_a^b \Big(f(x) - g(x)\Big)\,dx.$$

1. Compute the area of the region between the curves $y = x$ and $y = 6 - x^2$.

2. (Clicker) True or False: If $f(x)$ and $g(x)$ are continuous, then the area of the region R bounded by the curves $y = f(x)$ and $y = g(x)$, $x = a$, and $x = b$ is

$$\text{Area of } R = \int_a^b \Big|f(x) - g(x)\Big|\,dx.$$

3. Express the following areas using integrals. Time-permitting, evaluate the integrals to compute the areas.

   a. The area of the region between $y = 9 - x^2$ and $y = 5$.

   b. The area of the region between $y = x$ and $y = 8 - x$ from $x = 2$ to $x = 3$.

c. The area of the region between $y = \frac{x}{x^2+1}$ and $y = \frac{x}{5}$.

Volume Rule. Let S be a solid lying between $x = a$ and $x = b$ with the area of its vertical cross sections (i.e. cross section perpendicular to the x-axis) given by $A(x)$. If $A(x)$ is continuous, then

$$\text{Volume of S} = \int_a^b A(x)\, dx.$$

next class

1. (clicker) Let S be the solid whose base is enclosed by $x = y^2$ and $x = 3$ and whose cross sections are squares perpendicular to the x-axis. Give a formula for the area of a vertical cross section of S at any x between 0 and 3:

   a. x

   b. x^2

   c. $4x$

   d. I don't know, but I want a point for clicking in!

2. Let S be a pyramid with a square base of area 4 ft^2 and a height of 12 ft. Find the volume of S by following these steps:

   a. Draw the pyramid with its top at the origin and its base at $x = 12$.

   b. Note that the vertical cross sections of S are always squares. Use similar triangles to determine the side length of the cross section at any x between 0 and 12.

   c. Use your answer in (b) to determine the area of a vertical cross section of S at x.

   d. Use the Volume Rule and your answer from c to compute the volume of the pyramid.