Sept. 12th: 6.1 Area and 6.2 Volume

Warm Up Exercises

1. Write the following areas using integrals. Time permitting, compute the areas.
   a. The area of the region between \( f(x) = \frac{1}{x} \) and \( y = 0 \) (the x-axis) from \( x = 2 \) to \( x = 3 \).

   b. The area of the region bounded by the curves \( y = x - 2 \), the x-axis, \( x = 1 \), and \( x = 3 \).

2. Compute the volumes of the following solids:
   a. Solid 1:

   b. Solid 2:

   \[ \text{area}(B) = \frac{3}{2}\pi \]

In-Class Exercises

**Area Rule.** If \( f(x) \) and \( g(x) \) are continuous and if \( f(x) \geq g(x) \) for all \( x \) in \([a, b]\), then the area of the region \( R \) bounded by the curves \( y = f(x) \) and \( y = g(x) \), \( x = a \), and \( x = b \) is

\[
\text{Area of } R = \int_{a}^{b} (f(x) - g(x)) \, dx.
\]

1. Compute the area of the region between the curves \( y = x \) and \( y = 6 - x^2 \).

2. (Clicker) True or False: If \( f(x) \) and \( g(x) \) are continuous, then the area of the region \( R \) bounded by the curves \( y = f(x) \) and \( y = g(x) \), \( x = a \), and \( x = b \) is

\[
\text{Area of } R = \int_{a}^{b} |f(x) - g(x)| \, dx.
\]

3. Express the following areas using integrals. Time-permitting, evaluate the integrals to compute the areas.
   a. The area of the region between \( y = 9 - x^2 \) and \( y = 5 \).

   b. The area of the region between \( y = x \) and \( y = 8 - x \) from \( x = 2 \) to \( x = 3 \).
c. The area of the region between \( y = \frac{x^2}{x^2 + 1} \) and \( y = \frac{5}{6} \).

**Volume Rule.** Let \( S \) be a solid lying between \( x = a \) and \( x = b \) with the area of its vertical cross sections (i.e. cross section perpendicular to the \( x \)-axis) given by \( A(x) \). If \( A(x) \) is continuous, then

\[
\text{Volume of } S = \int_a^b A(x) \, dx.
\]

1. (clicker) Let \( S \) be the solid whose base is enclosed by \( x = y^2 \) and \( x = 3 \) and whose cross sections are squares perpendicular to the \( x \)-axis. Give a formula for the area of a vertical cross section of \( S \) at any \( x \) between 0 and 3:
   a. \( x \)
   b. \( x^2 \)
   c. \( 4x \)
   d. I don’t know, but I want a point for clicking in!

2. Let \( S \) be a pyramid with a square base of area 4 ft\(^2\) and a height of 12 ft. Find the volume of \( S \) by following these steps:
   a. Draw the pyramid with its top at the origin and its base at \( x = 12 \).
   b. Note that the vertical cross sections of \( S \) are always squares. Use similar triangles to determine the side length of the cross section at any \( x \) between 0 and 12.
   c. Use your answer in (b) to determine the area of a vertical cross section of \( S \) at \( x \).
   d. Use the Volume Rule and your answer from c to compute the volume of the pyramid.