Warm-up Problems

1. (a) What is an even function? Draw an example of a graph of an even function. 
   (b) What is an odd function? Draw an example of a graph of an odd function.

2. Compute the integrals, if possible, using the given $u$-substitution.
   (a) $\int \frac{15}{3 - 2x} \, dx = -\frac{15}{2} \ln |3 - 2x| + C$
      $u = 3 - 2x$
      $du = -2 \, dx$

   (b) $\int \frac{1}{x^2 (1 + \frac{1}{x})} \, dx = \frac{x}{x + 1} + C$
      $u = 1 + \frac{1}{x}$
      $du = -\frac{1}{x^2} \, dx$

   (c) $\int (2x + 1)e^{x^2+2x+3} \, dx = e^{x^2+2x+3} + C$
      $u = x^2 + 2x + 3$
      $du = 2x + 2 \, dx$

   (d) $\int \sqrt{7x + 9} \, dx = \frac{2}{21} (7x + 9)^{3/2} + C$
      $u = 7x + 9$
      $du = 7 \, dx$

   (e) $\int x\sqrt{4-x} \, dx$
      $u = x$
      $du = dx$

   (f) $\int x\sqrt{4-x} \, dx = \frac{2}{5} (4-x)^{5/2} - \frac{8}{3} (4-x)^{3/2} + C$
      $u = 4 - x$
      $du = -\, dx$

3. [Clicker] Select the best substitution for the integral.

   $\int 3x^2 csc^2(x^3 + 1) \, dx$

   (a) $3x^2$
   (b) $csc^2(x^3 + 1)$
   (c) $x^3$
   (d) $x^3 + 1$

   **Solution:** If $u = x^3 + 1$, $du = 3x^2 \, dx$ and the integral transforms to
   $\int csc^2 u \, du = -\cot u + C = -\cot(x^3 + 1) + C$

Class Problems

4. Make the substitution $u = \ln x$ and select the integral below that is equal to

   $\int_1^e \frac{\ln x}{x} \, dx$
(a) $\int_{1}^{e} u \, du$

(b) $\int_{0}^{1} u \, du \, \text{Correct}$

(c) $\int u \, du$

(d) $\int_{0}^{e} u \, du$

(e) They are all equal

5. Compute the definite integrals with substitution in two ways:
   (1) change the limits to $u$-limits and use these $u$-limits and
   (2) keep the $x$-limits and make sure you change back to $x$.

(a) $\int_{1}^{2} x \sqrt{x - 1} \, dx = \frac{16}{15}$

(b) $\int_{0}^{3} \frac{1}{5x + 1} \, dx = \frac{\ln 18}{5} - \frac{\ln 3}{5}$

(c) $\int_{1}^{2} e^{1/x} \, dx = e - \sqrt{e}$

(d) $\int_{1}^{3} \frac{x}{1 + 2x} \, dx = 3$

(e) $\int_{0}^{1} \frac{1}{(1 + \sqrt{x})^4} \, dx = \frac{1}{6}$

6. Clicker Select the best substitution for the integral.

$\int_{-1}^{1} x^5 \sqrt{x^4 + 1} \, dx$

(a) $u = x^5$

(b) $u = x^4 + 1$

(c) $u = x$

(d) $u = \sqrt{x^4 + 1}$

(e) Help! I’m going to fail this on the test!

Solution: This is actually a trick question! (Can you believe it!?) The answer is that the integral is equal to 0, but substitution isn’t going to solve this question. None of the substitutions are useful.