Math 132: Discussion Session: Week 2

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and your answers to the specified questions. Turn **this paper** in at the end of class. You do not need to turn in the question page or your work.

Additional Instructions: It is okay if you do not completely finish all of the problems (especially the challenge problem), but you should solve most of the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

Group Members

Group Answers

5.1-5.2: Riemann Sums and Limits of Riemann Sums

1. a. $L_4 =$

   b. $M_5 =$

   c. i. $R_n =$

      ii. $\int_2^5 (6x^2 - 4)\ dx =$

   d. i. $R_n =$

      ii. $\int_0^2 (x^3 - x)\ dx =$

5.2: Definite Integrals as Signed Area

1. a. $\int_0^2 \sqrt{16 - 4x^2} dx =$

   b. $\int_{-2}^5 \Big(3 + x - 2|x|\Big) dx =$

5.3: Fundamental Theorem of Calculus Part 1

1. a. $F'(x) =$

   b. $G'(x) =$

2. $F(x) =$

Challenge Problem Answer

1. a. State the critical point(s) and whether f has a local max, local min, or neither at each one:

   b. State the inflection point(s) and how the concavity of f changes at each one:

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5.1-5.2: Riemann Sums and Limits of Riemann Sums

Recall the formulas

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

1. a. Let $f(x) = \cos^2(x)$. Compute L_4 on $[\frac{\pi}{6}, \frac{\pi}{2}]$.

   b. Let $f(x) = \ln x$. Compute M_5 on $[1, 3]$.

   c. Let $f(x) = 6x^2 - 4$ on $[2, 5]$.

      i. Compute a formula for R_n (the Riemann sum $\displaystyle\sum_{i=1}^{n} f(x_i^*)\Delta x$ using right endpoints as sample points) that doesn't contain a $\sum_{i=1}^{n}$.

      ii. Compute the definite integral $\displaystyle\int_2^5 (6x^2 - 4)\, dx$ by computing $\displaystyle\lim_{n\to\infty} R_n$.

   d. Let $f(x) = x^3 - x$ on $[0, 2]$.

      i. Compute a formula for R_n (the Riemann sum $\displaystyle\sum_{i=1}^{n} f(x_i^*)\Delta x$ using right endpoints as sample points) that doesn't contain a $\sum_{i=1}^{n}$.

      ii. Compute the definite integral $\displaystyle\int_0^2 (x^3 - x)\, dx$ by computing $\displaystyle\lim_{n\to\infty} R_n$.

5.2: Definite Integrals as Signed Area

1. Compute the following definite integrals by interpreting them as areas

   a. $\displaystyle\int_0^2 \sqrt{16 - 4x^2}dx$

   b. $\displaystyle\int_{-2}^5 \Big(3 + x - 2|x|\Big)dx$. (Hint: First split the integral into three integrals)

5.3: Fundamental Theorem of Calculus Part 1

1. Using the Fundamental Theorem of Calculus Part 1, compute the derivatives of the following functions:

   a. $F(x) = \displaystyle\int_2^{e^{x^2}} \frac{x-1}{x+1}\, dx$

   b. $G(x) = \displaystyle\int_{\sin x}^{x^2} \ln(x+3)\, dx$.

2. Using the Fundamental Theorem of Calculus Part 1, give an antiderivative $F(x)$ of $f(x) = \sin^2(x) + e^{x^2}$ satisfying $F(2) = 0$. (Hint: Your answer can involve an integral.)

Challenge Problem

1. Let $F(x) = \int_0^x (t^2 - 5t + 6)\, dt$.

   a. Find the critical points of F (i.e. the points where $F'(x) = 0$) and determine whether they are local minima or local maxima.

   b. Find the points of inflection of F (i.e. the points where $F''(x) = 0$) and determine whether the concavity changes from up to down or from down to up at each one.