Warm-up Problems

1. **Clicker** Suppose you know the following \( \int_1^5 f(x) \, dx = 4 \), \( \int_3^5 f(x) \, dx = -7 \). Find \( \int_1^3 f(x) \, dx \)
   
   (a) \(-3\)
   
   (b) \(2\)
   
   (c) \(11\) **Correct**
   
   (d) \(13\)
   
   (e) \(\infty\)
   
   (Hint: Draw the areas)

2. Suppose \( f(x) = \int_1^x 3 \, dx \). Using areas, find
   
   (a) \( f(1) = 0 \)
   
   (b) \( f(2) = 3 \)
   
   (c) \( f(10) = 27 \)

3. Chain Rule Practice. Remember that \( [f(g(x))]' = f'(g(x)) \cdot g'(x) \)
   Suppose you have a function \( F(x) \) and you know \( F'(x) = \sin(x^2 + 1) \). Find
   
   (a) \( [F(x)]' = \sin(x^2 + 1) \)
   
   (b) \( [F(x^2)]' = \sin(x^4 + 1) \cdot 2x \)
   
   (c) \( [F(\sin x)]' = \sin(\sin^2 x + 1) \cdot \cos x \)
   
   (d) \( [F(\ln x)]' = \sin((\ln x)^2 + 1) \cdot \frac{1}{x} \)

Class Problems

**Properties of the Definite Integral**

I. \( \int_a^b c \, dx = c(b - a) \)
   
   IV. \( \int_a^b f(x) \, dx = -\int_b^a f(x) \, dx \)

II. \( \int_a^b f(x) + g(x) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx \)
   
   V. \( \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx \)

III. \( \int_a^b c f(x) \, dx = c \int_a^b f(x) \, dx \)

4. Suppose \( \int_1^b f(x) \, dx = 1 + 2b \). Using properties of integrals, compute:
   
   (a) \( \int_1^5 f(x) \, dx = 11 \)
   
   (b) \( \int_1^{1/2} f(x) \, dx = -2 \)
(c) \[ \int_1^6 3f(x) - 4 \, dx = 19 \]

(d) \[ \int_3^5 f(x) \, dx = 4 \]

5. Find a function with the given derivative:

(a) \[ f'(x) = \sin(x^2), \quad f(x) = \int_0^x \sin(t^2) \, dt \]

(b) \[ f'(x) = e^{x^2}, \quad f(x) = \int_{-2}^x e^{t^2} \, dt \]

(c) \[ f'(x) = e^{\cos x}, \quad f(x) = \int_1^x e^{\cos x} \, dt \]

6. Find the derivatives (compare to Problem 3):

(a) \[ \frac{d}{dx} \int_0^x \sin(t^2 + 1) \, dt = \sin(x^2 + 1) \]

(b) \[ \frac{d}{dx} \int_0^{x^2} \sin(t^2 + 1) \, dt = \sin(x^4 + 1) \cdot 2x \]

(c) \[ \frac{d}{dx} \int_0^{\sin x} \sin(t^2 + 1) \, dt = \sin(x^2 + 1) \cdot \cos x \]

(d) \[ \frac{d}{dx} \int_0^{\ln x} \sin(t^2 + 1) \, dt = \sin((\ln x)^2 + 1) \cdot \frac{1}{x} \]

7. **Clicker** Find the derivative

\[ \frac{d}{dx} \int_x^{x^2} \sin(t^2) \, dt \]

(a) \[ \sin(x^2) \cdot 2x \]

(b) \[ \sin(x^4) \cdot 2x \]

(c) \[ \sin(x^2) - \sin(x) \]

(d) \[ \sin(x^4) \cdot 2x - \sin(x^2) \]

(e) There is no derivative because this problem self destructs