Math 132 - September 2, 2016
Solutions

Warm-up Problems

1. $\boxed{\textbf{Clicker}}$ Suppose you know the following $\int_1^5 f(x)\,dx = 4$, $\int_3^5 f(x)\,dx = -7$. Find $\int_1^3 f(x)\,dx$

   (a) -3
   (b) 2
   (c) 11 $\boxed{\textbf{Correct}}$
   (d) 13
   (e) ∞

   (Hint: Draw the areas)

2. Suppose $f(x) = \int_1^x 3\,dx$. Using areas, find

   (a) $f(1) = 0$
   (b) $f(2) = 3$
   (c) $f(10) = 27$

3. Chain Rule Practice. Remember that $[f(g(x))]' = f'(g(x))\,g'(x)$
   Suppose you have a function $F(x)$ and you know $F'(x) = \sin(x^2+1)$. Find

   (a) $[F(x)]' = \sin(x^2+1)$
   (b) $\left[F(x^2)\right]' = \sin(x^4+1)\cdot 2x$
   (c) $[F(\sin x)]' = \sin(\sin^2 x+1)\cdot\cos x$
   (d) $[F(\ln x)]' = \sin((\ln x)^2+1)\cdot\frac{1}{x}$

Class Problems

Properties of the Definite Integral

I. $\int_a^b c\,dx = c(b-a)$

II. $\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

III. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

IV. $\int_a^b f(x);dx = -\int_b^a f(x)\,dx$

V. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

4. Suppose $\int_1^b f(x)\,dx = 1+2b$. Using properties of integrals, compute:

   (a) $\int_1^5 f(x)\,dx = 11$
   (b) $\int_{1/2}^1 f(x)\,dx = -2$

(c) $\int_1^6 3f(x) - 4\, dx = 19$

(d) $\int_3^5 f(x)\, dx = 4$

5. Find a function with the given derivative:

(a) $f'(x) = \sin(x^2), \qquad f(x) = \int_0^x \sin(t^2)\, dt$

(b) $f'(x) = e^{x^2}, \qquad f(x) = \int_{-2}^x e^{t^2}\, dt$

(c) $f'(x) = e^{\cos x}, \qquad f(x) = \int_1^x e^{\cos x}\, dt$

6. Find the derivatives (compare to Probelm 3):

(a) $\frac{d}{dx}\int_0^x \sin(t^2+1)\, dt = \sin(x^2+1)$

(b) $\frac{d}{dx}\int_0^{x^2} \sin(t^2+1)\, dt = \sin(x^4+1)\cdot 2x$

(c) $\frac{d}{dx}\int_0^{\sin x} \sin(t^2+1)\, dt = \sin(\sin^2 x+1)\cdot \cos x$

(d) $\frac{d}{dx}\int_0^{\ln x} \sin(t^2+1)\, dt = \sin((\ln x)^2+1)\cdot \frac{1}{x}$

7. $\boxed{\textbf{Clicker}}$ Find the derivative

$$\frac{d}{dx}\int_x^{x^2} \sin(t^2)\, dt$$

(a) $\sin(x^2)\cdot 2x$

(b) $\sin(x^4)\cdot 2x$

(c) $\sin(x^2) - \sin(x)$

(d) $\sin(x^4)\cdot 2x - \sin(x^2)$

(e) There is no derivative because this problem self destructs