Fundamental Theorem of Calculus (FTC). Let f be continuous on $[a, b]$. Then:

- Part 1: If $G(x) = \int_a^x f(t)\, dt$, then G is an antiderivative of f (i.e. $G'(x) = f(x)$).
- Part 2: If F is any antiderivative of f, then $\int_a^b f(t)\, dt = F(b) - F(a)$.

Class-time Exercises

1. Compute the derivatives of the following functions using the FTC Part 1.

   a. $G(x) = \int_0^x (t^5 - 9t^3)\, dt$

   b. $F(x) = \int_{x^2}^{x^4} \sqrt{t}\, dt$

   c. $H(x) = \int_{-6}^{\cos x} t^4\, dt$

   d. $I(x) = \int_{\sqrt{x}}^{x^2} \tan t\, dt$

2. (Clicker) Which of the following is an antiderivative of $\ln x$?

   a. $G(x) = \int_x^2 \ln x\, dx$

   b. $F(x) = x \ln x + x$

   c. Neither of the above

   d. Both of the above

September 2nd: 5.3: The Fundamental Theorem of Calculus

Extra Practice Exercise

1. Compute $\int_0^4 (x+3)\ dx$ using the limit definition of the definite integral, i.e. write:

$$\int_0^4 (x+3)\ dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

and compute the right-hand-side by following these steps:

a. Break $[0,4]$ into n subintervals $[x_0, x_1], [x_1, x_2], \dots, [x_{n-1}, x_n]$ of equal length Δx. Write down a formula for Δx. Write down a formula for x_i that works for $i = 1, 2, \dots, n$.

b. Select sample points $x_1^*, \dots, x_n^*$, one coming from each of the n subintervals. Specifically, for $i = 1, 2, \dots, n$ choose $x_i^* = x_i$ from (a) and then write down a formula for $f(x_i^*)$.

c. Plug your answers from (*a*) and (*b*) into the sum: $\sum_{i=1}^{n} f(x_i^*)\Delta x = \sum_{i=1}^{n}$

d. Using algebra and the formula $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, simplify your formula from (*c*) so that it no longer contains a $\sum_{i=1}^{n}$.

e. Compute $\int_0^4 (x+3)\ dx$ by computing $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$ using your formula from (*d*).

Warm-Up Exercise

2. (Clicker) Define $G(x) = \int_0^x f(t)\ dt$, where $y = f(t)$ is graphed below. Using the area interpretation of the definite integral, compute $G(0)$, $G(1)$, $G(2)$, $G(3)$. Which is largest?

a. $G(0)$

b. $G(1)$

c. $G(2)$

d. $G(3)$