Fundamental Theorem of Calculus (FTC). Let \( f \) be continuous on \([a, b]\). Then:

- Part 1: If \( G(x) = \int_a^x f(t) \, dt \), then \( G \) is an antiderivative of \( f \) (i.e. \( G'(x) = f(x) \)).

- Part 2: If \( F \) is any antiderivative of \( f \), then \( \int_a^b f(t) \, dt = F(b) - F(a) \).

**Class-time Exercises**

1. Compute the derivatives of the following functions using the FTC Part 1.
   a. \( G(x) = \int_0^x (t^5 - 9t^3) \, dt \)
   b. \( F(x) = \int_{x^2}^{x^4} \sqrt{t} \, dt \)
   c. \( H(x) = \int_{-6}^{\cos x} t^4 \, dt \)
   d. \( I(x) = \int_{\sqrt{x}}^{x^2} \tan t \, dt \)

2. (Clicker) Which of the following is an antiderivative of \( \ln x \)?
   a. \( G(x) = \int_2^x \ln x \, dx \)
   b. \( F(x) = x \ln x + x \)
   c. Neither of the above
   d. Both of the above
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Extra Practice Exercise

1. Compute \( \int_0^4 (x + 3) \, dx \) using the limit definition of the definite integral, i.e. write:

\[
\int_0^4 (x + 3) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x
\]

and compute the right-hand-side by following these steps:

a. Break \([0, 4]\) into \(n\) subintervals \([x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n]\) of equal length \(\Delta x\). Write down a formula for \(\Delta x\). Write down a formula for \(x_i^*\) that works for \(i = 1, 2, \ldots, n\).

b. Select sample points \(x_1^*, \ldots, x_n^*\), one coming from each of the \(n\) subintervals. Specifically, for \(i = 1, 2, \ldots, n\) choose \(x_i^* = x_i\) from (a) and then write down a formula for \(f(x_i^*)\).

c. Plug your answers from (a) and (b) into the sum:

\[
\sum_{i=1}^{n} f(x_i^*) \Delta x = \sum_{i=1}^{n}
\]

d. Using algebra and the formula \(\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\), simplify your formula from (c) so that it no longer contains a \(\sum_{i=1}^{n}\).

e. Compute \(\int_0^4 (x + 3) \, dx\) by computing \(\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x\) using your formula from (d).

Warm-Up Exercise

2. (Clicker) Define \(G(x) = \int_0^x f(t) \, dt\), where \(y = f(t)\) is graphed below. Using the area interpretation of the definite integral, compute \(G(0), G(1), G(2), G(3)\). Which is largest?

a. \(G(0)\)

b. \(G(1)\)

c. \(G(2)\)

d. \(G(3)\)