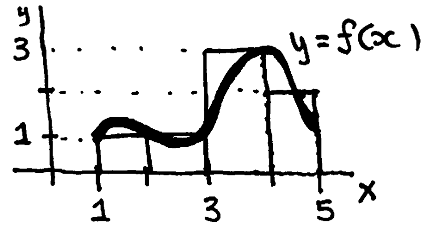
Answer key

September 1: 5.1/5.2: The Definite Integral

Warm-Up Exercises

1. Estimate the area under the graph of $y = f(x)$ by adding up the areas of the approximating rectangles.



area ≈ 1+1+3+2
= 7

2. Compute the sum: $\sum_{i=1}^{5} i = 1+2+3+4+5.$ $= 15$

3. Compute the sum: $\sum_{i=1}^{6} i^2.$ $= 1+4+9+16+25+36 = 91$

Class-time Exercises

1. Estimate the area under $y = \frac{1}{x}$ between $x = 1$ and $x = 3$ using 4 approximating rectangles. See notes from class

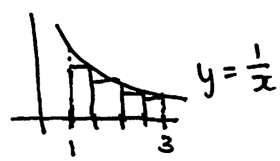
2. (Clicker) In the above problem, which is smallest?

   a. L_4

   b. R_4

   c. L_8

   d. R_8



all R_n's underestimate.
The fewer rectangles, the bigger the underestimate

3. Compute the following integrals by interpreting them as areas:

   a. $\int_{-2}^{2} \sqrt{4-x^2}\, dx.$ $= \frac{1}{2}(4\pi) = 2\pi$

   b. (Clicker) $\int_{-1}^{2} |x|\, dx.$

      a. 3

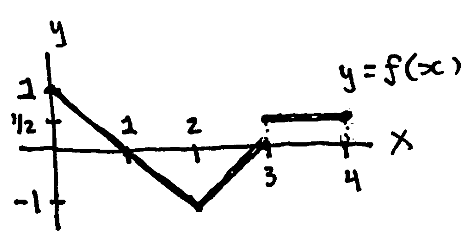
      b. 2.5

      c. 2

      d. 1.5

   c. $\int_{0}^{4} f(x)\, dx$, where the graph of $y = f(x)$ is provided.

   $= \frac{1}{2} - 1 + \frac{1}{2} = 0$



4. Use the limit definition of the definite integral to compute $\int_{0}^{2} x^2 - 3\, dx.$ See notes from class

Properties of the Definite Integral.

I. $\int_a^b c\,dx = c(b-a)$, where c is any constant.

II. $\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx.$

III. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, where c is any constant.

IV. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$

V. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx.$

5. Assume f is an integrable function satisfying

$$\int_1^b f(x)\,dx = 1 - b^{-1}, \qquad \text{for all } b > 0.$$

Using the properties of the definite integral, compute the following integrals

a. $\int_1^5 f(x)\,dx = 1 - 1/5$

b. $\int_{1/2}^1 f(x)\,dx = -\int_1^{1/2} f(x)\,dx = -\left[1 - 1/{1/2}\right] = -[1-2] = 1$

c. $\int_1^6 (3f(x)-4)\,dx = 3\int_1^6 f(x)\,dx - 4(6-1) = 3[1-1/6] - 20 = -1/2 - 17$

d. $\int_3^5 f(x)\,dx = \int_1^5 f(x)\,dx - \int_1^3 f(x)\,dx = (1-1/5) - (1-1/3) = 1/3 - 1/5$