Math 132: Discussion Session: Week 1

Directions: In groups of 3-4 students, work the problems on the following page. Below, list the members of your group and your answers to the specified questions. Turn this paper in at the end of class. You do not need to turn in the question page or your work.

Additional Instructions: It is okay if you do not completely finish all of the problems (especially the challenge problem), but you should solve most of the problems. Also, each group member should work through each problem, as similar problems may appear on the exam.

Group Members

Group Answers

Derivative Practice

1. a. \( f'(x) = \)
   
   b. \( g'(x) = \)
   
   c. \( h'(x) = \)
   
   d. \( i'(x) = \)

4.9: Practice Problem Answers

1. a. \( f(x) = \)
   
   b. \( f(x) = \)
   
   c. \( f(x) = \)

2. The car was traveling at a speed of

5.1: Introduction Problem Answer

1. d. The approximation to the area is

Challenge Problem Answer

1. a. The maximum distance is:
   
   b. The maximum distance is:
   
   c. The minimum time is:
   
   d. The distance between the stations is:
Math 132 Discussion Session: Week 1

Derivative Practice

1. Compute the derivatives of the following functions. You do not need to simplify your answer.

   a. \( f(x) = \frac{x^2 + 2}{x^3 + 4} \)

   b. \( g(x) = \arccos\left(\frac{1}{t}\right) - \text{arcsec}(t) \)

   c. \( h(x) = \ln\left(\frac{\cos(1 + x)}{1 + \cos(x)}\right) \)

   d. \( i(x) = xe^x \)

4.9: Practice Problems

1. In each problem below, find a function \( f \) satisfying the following properties:

   a. \( f'(x) = \sec x \sec x + \tan x \), \( f(\pi/4) = -1 \).

   b. \( f''(x) = x^2 + \frac{1}{x^2} \), \( f(2) = 3 \), \( f'(1) = 2 \).

   c. \( f'''(x) = \cos x \), \( f(0) = 1 \), \( f'(0) = 2 \), \( f''(0) = 3 \).

2. A car braked with a constant deceleration of 16 ft/s², producing skid marks measuring 200 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

5.1: Introduction Problem

1. Goal: Use 4 rectangles to approximate the area under the graph of \( f(x) = 1 + x^2 \) from \( x = -1 \) to \( x = 1 \) by following the steps below:

   a. Graph \( f(x) = 1 + x^2 \) from \( x = -1 \) to \( x = 1 \).

   b. Split \([-1, 1]\) into 4 subintervals of equal length (i.e. length \( \frac{1}{2} \)). Let \( x_1, x_2, x_3, x_4 \) denote the right endpoints of the four subintervals. Using this information, finish filling out the table below:

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \text{Subinterval } i )</th>
<th>( \text{Right-endpoint } x_i )</th>
<th>( f(x_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>([-1, -.5])</td>
<td>-.5</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. On your picture from (a), graph the four rectangles whose bases are your 4 subintervals from (b) and whose heights are the function’s values at the right endpoints.

d. Compute the areas of the rectangles from (c) and add them up to approximate the area under the graph of \( f(x) = 1 + x^2 \) from \( x = -1 \) to \( x = 1 \). Does this overestimate or underestimate the area?

e. In (d), you used rectangles whose heights are the function’s values at the right endpoints. If you used left endpoints instead of right endpoints, how would your answer change?
Challenge Problem

1. A high-speed bullet train accelerates and decelerates at the rate of $4 \text{ ft/s}^2$. Its maximum cruising speed is $90 \text{ mi/h}$.
   
a. What is the maximum distance the train can travel if it accelerates from rest until it reaches its cruising speed and then runs at that speed for 15 minutes?

b. Suppose that the train starts from rest and must come to a complete stop in 15 minutes. What is the maximum distance it can travel under those circumstances?

c. Find the minimum time that the train takes to travel between two consecutive stations that are 45 miles apart.

d. The trip from one station to the next takes 37.5 minutes. How far apart are the stations?