Trigonometric Substitution
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The point of trigonometric substitutions are to handle integrals with an integrand that contains expressions of the form:

$$\sqrt{a^2 - x^2} \qquad \sqrt{a^2 + x^2} \qquad \sqrt{x^2 - a^2}$$

To handle these we make a substitution to take advantage of trigonometric identities. You can also make *hyperbolic* trigonometric substitutions which take advantage of the hyperbolic trigonometric identities. (You will not be required to know the hyperbolic trigonometric substitutions, but I thought you might be interested in knowing that it is possible.)

Here is a table of when to use what substitution and what trigonometric substitution to use:

Integrand	Trig Identity	Substitution
$\sqrt{a^2 - x^2}$	$1 - \sin^2\theta = \cos^2\theta$	$x = a\sin\theta$
$\sqrt{a^2 + x^2}$	$1 + \tan^2\theta = \sec^2\theta$	$x = a\tan\theta$
$\sqrt{x^2 - a^2}$	$\sec^2\theta - 1 = \tan^2\theta$	$x = a\sec\theta$

Here is the general strategy:

1. Determine which substitution to make.
2. Make the substitution (and convert the integral to an integral in θ).
3. Integrate (anti differentiate).
4. Convert back to x's.

We will work some of these through in class.