Warm-Up Problems and Lecture Problems
March 26, 2003

1. True or false (properties about probability density functions). Make sure you justify your answer. If the answer if "false," try to turn the statement into a true statement.

   (a) If $f(t)$ is a probabilty density function, then $f(t) > 0$ for all t.

   (b) If $f(t)$ is a probability density function, then $\int_0^\infty f(t)\, dt = 1$.

   (c) If $f(t)$ is a pdf, it is possible that $\int_0^\infty f(t)\, dt = 0$.

   (d) If $F(t)$ is a cumulative distribution function, then $\lim_{t\to\infty} F(t) = 0$.

   (e) If $F(t)$ is a cdf, then $\lim_{t\to-\infty} F(t) = 0$.

   (f) If $f(t)$ is a pdf, then $\lim_{t\to\infty} f(t) = 1$.

   (g) Suppose X is a continuous random variable that we model with an exponential distribution. Then $P(X \leq 0) = 0$.

   (h) Suppose X is a continuous random variable that we model with an exponential distribution. Then $P(X \geq 0) = 1$.

   (i) Suppose X is a continuous random variable that we model with an exponential distribution. Suppose also that the mean of this random variable is 10. Then the pdf representing X is

   $$f(t) = \begin{cases} 0 & t < 0 \\ 10e^{-10t} & t \geq 0 \end{cases}$$

   (j) If $f(t)$ is a pdf, then the mean of the distribution is given by $\int_{-\infty}^\infty f(t)\, dt$.

Lecture Problems

2. Find the limits of the following sequences. (Hint, if you can't figure this out algebraically, try plugging in large values of n into the formula.)

   (a) $a_n = \frac{n}{n+1}$

   (b) $a_n = \left(\frac{2}{3}\right)^n$

   (c) $a_n = \left(-\frac{2}{3}\right)^n$

   (d) $a_n = (-1)^n$

   (e) $a_n = 3n^2$

   (f) $a_n = \frac{n}{n^2+1}$

   (g) $a_n = \frac{\ln n}{n}$

   (h) $a_n = \sqrt[n]{n}$

3. Determine which of the sequences from problem 2 are increasing, decreasing, monotonic or none of these.

4. Determine which of the sequences from problem 2 are bounded above, bounded below or bounded.