Warm-Up Problems and Lecture Problems
Solutions
March 26, 2003

1. True or false (properties about probability density functions). Make sure you justify your answer. If the answer if "false," try to turn the statement into a true statement.

   (a) If $f(t)$ is a probabilty density function, then $f(t) > 0$ for all t.
   Solution: This is is false. There are many pdf's that have $f(t) = 0$ for some values of t. Here's a true statement:

   If $f(t)$ is a pdf, then $f(t) \geq 0$ for all t.

   (b) If $f(t)$ is a probability density function, then $\int_0^\infty f(t)\,dt = 1$.
   Solution: False. Here is a true statement:

   If $f(t)$ is a pdf, then $\int_{-\infty}^\infty f(t)\,dt = 1$.

   (c) If $f(t)$ is a pdf, it is possible that $\int_0^\infty f(t)\,dt = 0$.
   Solution: This is true. See if you can find an example of such a pdf.

   (d) If $F(t)$ is a cumulative distribution function, then $\lim_{t\to\infty} F(t) = 0$.
   Solution: False. Here is a true statement:

   $\lim_{t\to\infty} F(t) = 1$

   (e) If $F(t)$ is a cdf, then $\lim_{t\to-\infty} F(t) = 0$.
   Solution: True.

   (f) If $f(t)$ is a pdf, then $\lim_{t\to\infty} f(t) = 1$.
   Solution: False. The true statement should be $\lim_{t\to\infty} f(t) = 0$.

   (g) Suppose X is a continuous random variable that we model with an exponential distribution. Then $P(X \leq 0) = 0$.
   Solution: True.

   (h) Suppose X is a continuous random variable that we model with an exponential distribution. Then $P(X \geq 0) = 1$.
   Solution: True.

   (i) Suppose X is a continuous random variable that we model with an exponential distribution. Suppose also that the mean of this random variable is 10. Then the pdf representing X is

   $$f(t) = \begin{cases} 0 & t < 0 \\ 10e^{-10t} & t \geq 0 \end{cases}$$

   Solution: False. The correct pdf should be:

   $$f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{10}e^{-t/10} & t \geq 0 \end{cases}$$

   (j) If $f(t)$ is a pdf, then the mean of the distribution is given by $\int_{-\infty}^\infty f(t)\,dt$.
   Solution: False. The integral given will always be equal to 1. The average is given by $\int_{-\infty}^\infty tf(t)\,dt$.

Lecture Problems

2. Find the limits of the following sequences. (Hint, if you can't figure this out algebraically, try plugging in large values of n into the formula.)

   (a) $a_n = \frac{n}{n+1}$
   Solution: $\lim_{n\to\infty} a_n = 1$

   (b) $a_n = \left(\frac{2}{3}\right)^n$
   Solution: $\lim_{n\to\infty} a_n = 0$

   (c) $a_n = \left(-\frac{2}{3}\right)^n$
   Solution: $\lim_{n\to\infty} a_n = 0$
   (To do this, compute $\lim_{n\to\infty} |a_n|$).

   (d) $a_n = (-1)^n$
   Solution: This limit does not exist, it bounces between -1 and 1.

   (e) $a_n = 3n^2$
   Solution: $\lim_{n\to\infty} a_n = \infty$

   (f) $a_n = \frac{n}{n^2+1}$
   Solution: $\lim_{n\to\infty} a_n = 0$
   (Use L'Hopital's rule, for example.)

   (g) $a_n = \frac{\ln n}{n}$
   Solution: $\lim_{n\to\infty} a_n = 0$
   (This is another good candidate for L'Hopital's rule)

   (h) $a_n = \sqrt[n]{n}$
   Solution: $\lim_{n\to\infty} a_n = 1$
   Yet another good one for L'Hopital's rule. Here is the work, notice the use of L'Hopital at the last line.

$$
\begin{aligned}
\lim_{n\to\infty} \sqrt[n]{n} &= \lim_{n\to\infty} n^{1/n} = \lim_{n\to\infty} e^{\ln n^{1/n}} \\
&= \lim_{n\to\infty} e^{(1/n)\ln n} \\
&= e^{\lim_{n\to\infty}(1/n)\ln n} = e^0 = 1
\end{aligned}
$$

3. Determine which of the sequences from problem 2 are increasing, decreasing, monotonic or none of these.
   Solution:

Increasing	2a, 2e
Decreasing	2b, 2f
Monotonic	2a, 2b, 2e, 2f
Not Monotonic	2c, 2d, 2g, 2h

4. Determine which of the sequences from problem 2 are bounded above, bounded below or bounded.
   Solution:

Bounded above	2a, 2b, 2c, 2d, 2f, 2g, 2h
Bounded below	2a, 2b, 2c, 2d, 2e, 2f, 2g, 2h
Bounded	2a, 2b, 2c, 2d, 2f, 2g, 2h