Warm-Up Problems and Lecture Problems
Solutions
March 24, 2003

1. Suppose you have a continuous random variable X that has a probability density function given as:

$$f(t) = \begin{cases} \frac{1}{10} & 4 \leq t \leq 14 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What are the two conditions a function has to satisfy in order to be a probability density function? Show that $f(t)$ satisfies these.
   Solution: First we need $f(t) \geq 0$ for all t and this is obviously true. We also need $\int_{-\infty}^{\infty} f(t)\,dt = 1$ and this is pretty easy to check too:

$$\begin{aligned} \int_{-\infty}^{\infty} f(t)\,dt &= \int_{-\infty}^{4} f(t)\,dt + \int_{4}^{14} f(t)\,dt + \int_{14}^{\infty} f(t)\,dt \\ &= \int_{-\infty}^{4} 0\,dt + \int_{4}^{14} \frac{1}{10}\,dt + \int_{14}^{\infty} 0\,dt = 1 \end{aligned}$$

   (b) What is the probability that $10 \leq X \leq 15$?
   Solution: We just integrate:

$$\begin{aligned} P(10 \leq X \leq 15) &= \int_{10}^{15} f(t)\,dt = \int_{10}^{14} f(t)\,dt + \int_{14}^{15} f(t)\,dt \\ &= \int_{10}^{14} \frac{1}{10}\,dt + \int_{14}^{15} 0\,dt = \frac{4}{10} = 0.4 \end{aligned}$$

2. Suppose you are a used car deal and you have sold 10 cars. For these cars, they all came back in a certain number of weeks with problems. Here is how many weeks it took each car to come back:

Car Number:	1	2	3	4	5	6	7	8	9	10
Weeks:	50	102	243	12	323	72	2	45	21	13

   Find the average number of weeks before the car came back.
   Solution: Just add up and divide by 10:

$$\text{Average} = \frac{50 + 102 + 243 + 12 + 323 + 72 + 2 + 45 + 21 + 13}{10} = 88.3$$

Lecture Problems

3. Find the cumulative distribution function for problem 1. We will do this in steps. We will let $F(x)$ denote the CDF.

   (a) What is the definition of the CDF?
   Solution:

   $$F(x) = \int_{-\infty}^{x} f(t)\ dt$$

   (b) If $x < 4$, what is $F(x)$?
   Solution:

   $$F(x) = \int_{-\infty}^{x} f(t)\ dt = \int_{-\infty}^{x} 0\ dt = 0$$

   (c) If $4 \leq x \leq 14$, then what is $F(x)$?
   Solution:

   $$\begin{aligned} F(x) &= \int_{-\infty}^{x} f(t)\ dt = \int_{-\infty}^{4} f(t)\ dt + \int_{4}^{x} f(t)\ dt \\ &= \int_{-\infty}^{4} 0\ dt + \int_{4}^{x} \frac{1}{10}\ dt = \frac{x-4}{10} \end{aligned}$$

   (d) If $x > 14$ then what is $F(x)$?
   Solution:

   $$\begin{aligned} F(x) &= \int_{-\infty}^{x} f(t)\ dt = \int_{-\infty}^{4} f(t)\ dt + \int_{4}^{14} f(t)\ dt + \int_{14}^{x} f(t)\ dt \\ &= \int_{-\infty}^{4} 0\ dt + \int_{4}^{14} \frac{1}{10}\ dt + \int_{14}^{x} 0\ dt \\ &= 0 + 1 + 0 = 1 \end{aligned}$$

   (e) What is the CDF, $F(x)$?
   Solution:

   $$F(x) = \begin{cases} 0 & x < 4 \\ \frac{x-4}{10} & 4 \leq x \leq 14 \\ 1 & x \geq 14 \end{cases}$$

4. Suppose you want to model your used car failure with an exponential probability density function:

$$f(t) = \begin{cases} 0 & t < 0 \\ ce^{-ct} & t \geq 0 \end{cases}$$

(a) What should c be using the data from problem 2?
Solution: We know that $c = \frac{1}{\mu}$ where μ is the average. So, $c = 1/88.3$.

(b) Using this exponential model, what is the probability that a car will fail within the first year of purchase?
Solution: We can do this integrating or using the cumulative distribution function:

$$\begin{aligned} P(X \leq 52) &= \int_{-\infty}^{52} f(t)\, dt = \int_{0}^{52} ce^{-ct}\, dt \approx 0.445 \\ &= F(52) - F(0) = (1 - e^{-52c}) - (1 - e^{0}) = 0.445 \end{aligned}$$