Warm-Up Problems and Lecture Problems
Solutions
March 17, 2003

1. (Review) Rotate the region in the plane bounded by $y = x^2$ and $y = 4x$ around the axis $y = -5$. Set up an integral representing the volume of this region.
   Solution:
$$V = \int_0^4 \pi(r_o^2 - r_i^2)\,dx = \pi \int_0^4 (5+4x)^2 - (5+x^2)^2\,dx = \frac{1216\pi}{5}$$

2. Sketch the parametric curve and find the length (at least set up the integral!).
   (a) $x = t^3/3$, $y = t^2/2$, $0 \le t \le 1$.
   (Remember that the formula for arc length is $L = \int_a^b \sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt$.)
   Solution:
$$L = \int_0^1 \sqrt{(t^2)^2 + (t)^2}\,dt = \int_0^1 t\sqrt{t^2+1}\,dt = \frac{1}{3}(2^{3/2} - 1)$$
   (b) $x = 4\sin t$, $y = 4\cos t - 5$, $0 \le t \le \pi$.
   Solution:
$$L = \int_0^\pi \sqrt{(4\cos t)^2 + (-4\sin t)^2}\,dt = \int_0^\pi 4\,dt = 4\pi$$

Lecture Problems

3. Set up the integral representing the length of the curve (try to compute the integral if you have time – keep in mind that you might have to use numerics).

   (a) $y = e^x$, from $(0, 1)$ to $(1, e)$.
   Solution:

   $$L = \int_0^1 \sqrt{1 + e^{2x}}\, dx \cong 2.0035$$

   (b) $y = \frac{x^3}{6} + \frac{1}{2x}$, $\frac{1}{2} \leq x \leq 1$.
   Solution:

   $$L = \int_{1/2}^1 \sqrt{1 + \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2}\, dx \cong 0.6458$$

   (c) $y = \sin x$, $0 \leq x \leq 2\pi$.
   Solution:

   $$L = \int_0^{2\pi} \sqrt{1 + \cos^2 x}\, dx \cong 3.8202$$

   (d) $x = y^4$, $-1 \leq y \leq 2$.
   Solution:

   $$L = \int_{-1}^2 \sqrt{1 + (4y^3)^2}\, dy \cong 18.2471$$