Warm-Up Problems and Lecture Problems
Solutions
March 14, 2003

1. Consider the triangle with vertices $(0,0)$, $(1,2)$ and $(1,0)$.

   (a) Find the equation of the line between $(0,0)$ and $(1,2)$.
   Solution: $y = 2x$.

   (b) Rotate this triangle about the line $y = 0$ (the x-axis) and set up an integral representing the volume of revolution. Draw the picture too!
   Solution:

   $$V = \int_0^1 \pi(2x)^2 \, dx$$

   (c) Rotate this triangle about the line $y = 4$ and set up an integral representing the volume. Draw the picture too!
   Solution:

   $$V = \int_0^1 \pi \left[4^2 - (4-2x)^2\right] \, dx$$

   (d) Rotate this triangle about the line $x = 0$ and set up an integral representing the volume. Draw the picture too!
   Solution:

   $$V = \int_0^2 \pi \left[1^2 - \left(\frac{1}{2}x\right)^2\right] \, dy$$

   (e) Rotate this triangle about the line $x = 5$ and set up an integral representing the volume. Draw the picture too!
   Solution:

   $$V = \int_0^2 \pi \left[\left(5 - \frac{1}{2}x\right)^2 - 4^2\right] \, dy$$

Lecture Problems

2. Sketch the graph of the parametric equation for $-2 \leq t \leq 3$:

$$x = t^2 - 2t \qquad y = t + 1$$

Solution: I made the table and graphed the points:

t	x	y
-2	8	-1
-1	3	0
0	0	1
1	-1	2
2	0	3
3	3	4

Here is an estimate of the length of this curve by using line segments:

t-interval	Endpoints of segement	Length of segment
$[-2,-1]$	$(8,-1),(3,0)$	$\sqrt{26}$
$[-1,0]$	$(3,0),(0,1)$	$\sqrt{10}$
$[0,1]$	$(0,1),(-1,2)$	$\sqrt{2}$
$[1,2]$	$(-1,2),(0,3)$	$\sqrt{2}$
$[2,3]$	$(0,3),(3,4)$	$\sqrt{10}$
		$\cong 14.2520$

Setting this up the length of this curve as an integral is:

$$\int_{-2}^{3} \sqrt{(2t-2)^2 + (1)^2}\, dt$$

3. Set up an integral representing the length of the curve. If you have time, try and sketch the curve.

$$x = t^2 \qquad y = e^t \qquad 1 \leq t \leq 4$$

Solution:

$$\int_{1}^{4} \sqrt{(2t)^2 + (e^t)^2}\, dt$$