Warm-Up Problems and Lecture Problems
Solutions
March 12, 2003

1. Consider the function $y = x^2$.

   (a) For a given x-value, consider the vertical line segment from the x-axis to the graph of $y = x^2$. Let $r_1(x)$ denote the length of this line segment. Find a nice expression for $r_1(x)$.
   Solution: The length of this line segment is the y-value, so $r_1(x) = x^2$.

   (b) For a given y-value, consider the horizontal line segment from the y-axis to the graph of $y = x^2$. Let $r_2(y)$ denote the length of this line segment. Find a nice expression for $r_2(y)$.
   Solution: The length of this line segment is the x-value, so $r_2(y) = \sqrt{y}$.

2. Find the area of an equilateral triangle with side length a.
   Solution: Draw the picture. We know the area of a triangle is $\frac{1}{2}bh$ where b is the length of a base and h is the length of the height (perpendicular to the base). So, you have to use a little geometry and the Pythagorean Theorem to see if the base has length a, then the height has length $\frac{\sqrt{3}}{2}a$. Therefore the area is $\frac{\sqrt{3}}{4}a^2$.

Lecture Problems

3. Take the region bounded by $y = e^x$, the x-axis, $y = 0$ and $y = 1$. Rotate the region around the x-axis and compute the volume.
   Here are the steps:

   (a) Draw the region.

   (b) For a given x-value, find the height of the vertical line segment from the x-axis to the curve $y = e^x$. (This will be the radius of the circular cross section).
   Solution: Height is equal to y, or e^x.

   (c) Find the area of a cross section ($A = \pi r^2$).
   Solution: $A(x) = \pi(e^x)^2 = \pi e^{2x}$.

   (d) Set up the integral representing the volume.
   Solution:

   $$\text{Vol} = \int_0^1 \pi e^{2x}\, dx$$

   (e) Compute the integral.

   $$\text{Vol} = \int_0^1 \pi e^{2x}\, dx = \frac{\pi}{2}(e^2 - 1) \cong 10.0359$$