Warm-Up Problems and Lecture Problems
Solutions
March 10, 2003

1. Solve the following equation for P:

$$\frac{M-P}{P} = Ae^{-kt} \tag{1}$$

Solution:

$$\begin{aligned} M - P &= PAe^{-kt} \\ M &= PAe^{-kt} + P \\ M &= (Ae^{-kt} + 1)P \\ P &= \frac{M}{Ae^{-kt} + 1} \end{aligned}$$

2. Suppose you know that in equation (1) that M and k are constants given by $M = 100$ and $k = 2$. Suppose you also know that $P = 20$ when $t = 0$. Find A. Using the previous problem, write P as a function of t.
Solution: To find A, we plug in all the numbers: $M = 100$, $k = 2$, $P = 20$ and $t = 0$:

$$\frac{100-20}{20} = Ae^0$$

Which gives us that $A = 4$. This gives the equation:

$$P = \frac{100}{4e^{-2t} + 1}$$

3. (Review problem) Suppose you use Simpson's Rule for the following integral: $\int_1^5 e^{3x}\,dx$.

(a) What is the error formula for Simpson's rule (and what does everything mean)?
Solution:

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$

(b) Find a good value for K for out integral.
Solution: $f(x) = e^{3x}$ so $f^{(4)}(x) = 3^4 e^{3x}$. For all x between 1 and 5, we have $|f^{(4)}(x)| \leq 3^4 e^{15}$ (draw the graph!).

(c) If $n = 100$, then what is the worst the error can be?
Solution: Just plug everything into the formula:

$$|E_S| \leq \frac{3^4 e^{15}(5-1)^5}{180 \cdot 100^4} = 15.06$$

(so, not too accurate)

(d) Find n so that the error is less then 0.01.
Solution: Solve the inequality:

$$\frac{3^4 e^{15}(5-1)^5}{180n^4} < 0.01$$

to get $n > 622.99$.

Lecture Problems

4. Consider the curve $y = x^2 + 1$.

   (a) Draw this curve and rotate it around the y-axis.

   (b) Find the area function $A(y)$ (the area of a y-cross section).
   Solution:

   $$A(y) = \pi x^2 = \pi(\sqrt{y-1})^2 = \pi(y-1)$$

   (c) Set up an integral representing the volume of the solid of revolution from $y = 2$ to $y = 4$. (What does this part of the solid look like?)
   Solution:

   $$V = \int_2^4 A(y)\, dy = \int_2^4 \pi(y-1)\, dy = 4\pi$$

5. Repeat the previous exercise, but rotate the curve around the x-axis and find the volume from $x = -1$ to $x = 3$.
   Solution:

   $$A(x) = \pi y^2 = \pi(x^2+1)^2$$

   $$V = \int_{-1}^3 \pi(x^2+1)^2 = \int_{-1}^3 \pi(x^4 + 2x^2 + 1) = \frac{1072\pi}{15}$$