Warm-Up Problems and Lecture Problems
Solutions
February 28, 2003

1. Solve the following equation for P:

$$\frac{M-P}{P} = Ae^{-kt} \tag{1}$$

Solution:

$$\begin{aligned} M-P &= PAe^{-kt} && (2)\\ M &= PAe^{-kt}+P && (3)\\ M &= (Ae^{-kt}+1)P && (4)\\ P &= \frac{M}{Ae^{-kt}+1} && (5) \end{aligned}$$

2. Suppose you know that in equation (1) that M and k are constants given by $M = 100$ and $k = 2$. Suppose you also know that $P = 20$ when $t = 0$. Find A. Using the previous problem, write P as a function of t.
Solution: To find A, we plug in all the numbers: $M = 100$, $k = 2$, $P = 20$ and $t = 0$:

$$\frac{100-20}{20} = Ae^{0}$$

Which gives us that $A = 4$. This gives the equation:

$$P = \frac{100}{4e^{-2t}+1}$$

Lecture Problems

3. Draw a slopefield and some possible trajectories for the logistic differential equation:

$$\frac{dP}{dt} = 0.1P\left(1 - \frac{P}{500}\right)$$

4. Draw a slopefield and some possible trajectories for the differential equation:

$$\frac{dP}{dt} = -0.1P\left(1 - \frac{P}{500}\right)$$

5. Draw a slopefield and some possible trajectories for the differential equation:

$$\frac{dP}{dt} = 0.1P\left(1 - \frac{P}{500}\right)\left(1 - \frac{P}{250}\right)$$