Warm-Up Problems and Lecture Problems
Solutions
February 26, 2003

1. Consider the following initial value problem:

$$\frac{dy}{dt} = \frac{10 - y}{7} \qquad y(0) = 21$$

(a) Find the *general* solution (i.e., don't take the initial value into account). Write you answer as a function of t (i.e., write your answer as $y =$).
Solution: First separate the variables:

$$\frac{dy}{10 - y} = \frac{dt}{7}$$

Now integrate: $-\ln|10 - y| = \frac{1}{7}t + C$. Now, multiply both sides by -1 and exponentiate both sides: $\ln|10 - y| = -\frac{1}{7}t + C$ and then:

$$|10 - y| = e^{-t/7 - C} = e^{-C}e^{-t/7}$$

Get rid of the absolute values and make $A = \pm e^{-C}$:

$$10 - y = Ae^{-t/7}$$

Finally, solve for y:

$$y = 10 - Ae^{-t/7}$$

(b) Find the specific solution.
Solution: Plug in the intial condition and solve for A:

$$21 = 10 - Ae^{0}$$

So, $A = -11$ and the solution is:

$$y = 10 + 11e^{-t/7}$$

Lecture Problems

2. In 1980, the population of Nepal was 14.6 million. In 1990, the population was 18.1 million. Let t represent years and let $t = 0$ correspond to 1980. Find a population model for the population of Nepal:

   $$P = P_0 e^{kt}$$

   (a) Find P_0.
   Solution: Plug $t = 0$ and $P = 14.6$ into the equation to get $P_0 = 14.6$.

   (b) Find k.
   Solution: We already know $P_0 = 14.6$, so now we plug in $t = 10$ and $P = 18.1$ and solve the equation: $18.1 = 14.6e^{10k}$. Divide by 14.6 and take the natural log of both sides to get $k = 0.0215$. Therefore, our model is:

   $$P = 14.6e^{0.0215t}$$

   (c) Using your model, predict the population in year 1997. (The actual population in 1997 was 22.59 Million.)
   Solution: This is a matter of plugging in 17 for t in our model to get $P = 21.04$ million.

   (d) Using your model, predict the population in year 2010.
   Solution: Now plug in $t = 30$ to get $P = 27.8$ million people.

3. (Nearly identical to the previous problem). In 1980, the population of Nepal was 14.6 million. In 1990, the population was 18.1 million. Let t represent years and let $t = 0$ correspond to 1900. Find a population model for the population of Nepal:

   $$P = P_0 e^{kt}$$

   (a) Find P_0 and k.
   Solution: We now have two equations and two unknowns to solve (by using our two data points):

   $$\begin{aligned} 14.6 &= P_0 e^{80k} \\ 18.1 &= P_0 e^{90k} \end{aligned}$$

   I take the first equation and solve for $P_0 = 14.6e^{-80k}$. I plug this in for P_0 in the second equation: $18.1 = 14.6e^{-80k}e^{90k}$, or $18.1 = 14.6e^{10k}$. I can now solve this as before to get $k = 0.0215$ (as before). I can now plug back in to get $P_0 = 2.61$. So, our model is:

   $$P = 2.61e^{0.0215t}$$

   (b) Using your model, predict the population in year 1997. (The actual population in 1997 was 22.59 Million.)
   Solution: We plug $t = 97$ into our model and get $P = 21.00$ million (the difference from the previous result is due to round off error).

   (c) Using your model, predict the population in year 2010.
   Solution: Plugging in $t = 110$ gives $P = 27.8$ million.