Warm-Up Problems and Lecture Problems
Solutions
February 17, 2003

1. Find the points of intersection for the following sets of graphs.

   (a) $y = 12 - x^2$ and $y = x^2$
   Solution: Solve the equation $12 - x^2 = x^2$ and get $x = \pm\sqrt{6}$. So the points of intersection are $(-\sqrt{6}, 6)$ and $(\sqrt{6}, 6)$.

   (b) $y = x^2$ and $y = 2x + 8$
   Solution: Solve the equation $x^2 = 2x + 8$ and get the points $(-2, 4)$ and $(4, 16)$.

   (c) $x = y^2$ and $x = y + 6$
   Solution: Solve the equation $y^2 = y + 6$ and get the points: $(4, -2)$ and $(9, 3)$.

   (d) $x = y^2 - 2y - 2$ and $x = 4 + y - 2y^2$
   Solution: Solve the equation $y^2 - 2y - 2 = 4 + y - 2y^2$ and get: $(1, -1)$ and $(-2, 2)$.

2. Using your calculator, estimate the points of intersection of the following sets of graphs:

   (a) $y = x^3$ and $y = 32 - x^4$
   Solution: You can graph these or use a polynomial solver on your calculator. I got the following points: $(-2.67388, -19.11726)$ and $(2.16284, 10.11749)$.

   (b) $y = \frac{7x}{(x^2+1)^{3/2}}$ and $y = \frac{x^2}{3}$
   Solution: By graphing you can find the points of intersection: $(0, 0)$ and $(1.96331, 1.28487)$.

Lecture Problems

3. Find the area between the curves (hint: see your work in problem 1)

   (a) $y = 12 - x^2$ and $y = x^2$
   Solution:

   $$\int_{-\sqrt{6}}^{\sqrt{6}} (12 - x^2) - (x^2)\, dx = 16\sqrt{6}$$

   (b) $y = x^2$ and $y = 2x + 8$
   Solution:

   $$\int_{-2}^{4} (2x + 8) - (x^2)\, dx = 36$$

   (c) $x = y^2$ and $x = y + 6$
   Solution:

   $$\int_{-2}^{3} (y + 6) - (y^2)\, dy = \frac{125}{6}$$

   (d) $x = y^2 - 2y - 2$ and $x = 4 + y - 2y^2$
   Solution:

   $$\int_{-1}^{2} (4 + y - 2y^2) - (y^2 - 2y - 2)\, dy = \frac{27}{2}$$

4. Find the area between the curves (hint: see your work in problem 2)

   (a) $y = x^3$ and $y = 32 - x^4$
   Solution:

   $$\int_{-2.67388}^{2.16284} (32 - x^4) - (x^3)\, dx = 125.28175$$

   (b) $y = \frac{7x}{(x^2+1)^{3/2}}$ and $y = \frac{x^2}{3}$
   Solution:

   $$\int_{0}^{1.96331} \frac{7x}{(x^2 + 1)^{3/2}} - \frac{x^2}{3}\, dx = 2.98210$$