

Warm-Up Problems and Lecture Problems
Solutions
February 14, 2003

1. Compute the following limits:

(a)

$$\lim_{t \rightarrow \infty} 25 - \frac{18}{t^3}$$

(b)

$$\lim_{x \rightarrow \infty} \frac{5x^3 - 2x}{16x^3 + 1}$$

(c)

$$\lim_{b \rightarrow \infty} x e^{-x}$$

(d)

$$\lim_{t \rightarrow \infty} \frac{x}{\ln x}$$

(e)

$$\lim_{t \rightarrow \infty} \ln t$$

Lecture Problems

2. Consider the function defined below:

$$G(t) = \int_0^t e^{-x} dx$$

- (a) Find a “simple” expression for $G(t)$ (i.e., without an integral).
(b) Fill out the following chart:

t	0	1	2	3	10
$G(t)$					

- (c) Find the limit:

$$\lim_{t \rightarrow \infty} G(t)$$

- (d) What does this limit correspond to in terms of area under the curve?

Solution: This means that the area under the curve $y = e^{-x}$ from 0 to ∞ is equal to 1.

3. Compute the following improper integrals. What makes these integrals “improper”? Are the integrals divergent or convergent?

(a)

$$\int_1^{\infty} \frac{1}{x^5} dx$$

(b)

$$\int_1^{\infty} \frac{1}{\sqrt{x}} dx$$

(c)

$$\int_1^{\infty} \frac{1}{x^{1/3}} dx$$

(d)

$$\int_1^{\infty} \frac{1}{x^{1.2}} dx$$

(e)

$$\int_0^1 \frac{1}{x^5} dx$$

(f)

$$\int_0^1 \frac{1}{\sqrt{x}} dx$$

(g)

$$\int_0^1 \frac{1}{x^{1/3}} dx$$

(h)

$$\int_0^1 \frac{1}{x^{1.2}} dx$$

4. Determine the values of p for which the following integral converges:

$$\int_1^{\infty} \frac{1}{x^p} dx$$

5. Determine the values of p for which the following integral converges:

$$\int_0^1 \frac{1}{x^p} dx$$