Warm-Up Problems and Lecture Problems
Solutions
February 14, 2003

1. Compute the following limits:

   (a)

   $$\lim_{t\to\infty} 25 - \frac{18}{t^3}$$

   Solution: 25

   (b)

   $$\lim_{x\to\infty} \frac{5x^3 - 2x}{16x^3 + 1}$$

   Solution: $\frac{5}{16}$

   (c)

   $$\lim_{b\to\infty} xe^{-x}$$

   Solution: 0. This is a nice place to use L'Hopital's rule:

   $$\lim_{b\to\infty} xe^{-x} = \lim_{b\to\infty} \frac{x}{e^x} = \lim_{b\to\infty} \frac{x}{e^x} = \lim_{b\to\infty} \frac{1}{e^x} = 0$$

   (d)

   $$\lim_{t\to\infty} \frac{x}{\ln x}$$

   Solution: ∞. Another nice use of L'Hopital's rule:

   $$\lim_{b\to\infty} \frac{x}{\ln x} = \lim_{b\to\infty} \frac{1}{1/x} = \lim_{b\to\infty} x = \infty$$

   (e)

   $$\lim_{t\to\infty} \ln t$$

   Solution: There are a couple ways to see that this limit is ∞. One is to think about $\ln t$ as the inverse function of e^t and use properties of e^t. Another way to think about this is to make a substitution $t = e^u$ and notice that $t \to \infty$ as $u \to \infty$ and therefore

   $$\lim_{t\to\infty} \ln t = \lim_{u\to\infty} \ln e^u = \lim_{u\to\infty} u = \infty$$

Lecture Problems

2. Consider the function defined below:

$$G(t) = \int_0^t e^{-x}\, dx$$

   (a) Find a "simple" expression for $G(t)$ (i.e., without an integral).
   Solution: : $G(t) = 1 - e^{-t}$

   (b) Fill out the following chart:

t	0	1	2	3	10
$G(t)$					

   Solution: :

t	0	1	2	3	10
$G(t)$	0	0.6321	0.8647	0.9502	0.999954

   (c) Find the limit:

$$\lim_{t\to\infty} G(t)$$

   Solution: 1.

   (d) What does this limit correspond to in terms of area under the curve?
   Solution: This means that the area under the curve $y = e^{-x}$ from 0 to ∞ is equal to 1.

3. Compute the following improper integrals. What makes these integrals "improper"? Are the integrals divergent or convergent?

   (a)
   $$\int_1^\infty \frac{1}{x^5}\,dx$$

   Solution: 1/4

   (b)
   $$\int_1^\infty \frac{1}{\sqrt{x}}\,dx$$

   Solution: Diverges

   (c)
   $$\int_1^\infty \frac{1}{x^{1/3}}\,dx$$

   Solution: Diverges

   (d)
   $$\int_1^\infty \frac{1}{x^{1.2}}\,dx$$

   Solution: 5

   (e)
   $$\int_0^1 \frac{1}{x^5}\,dx$$

   Solution: Diverges

   (f)
   $$\int_0^1 \frac{1}{\sqrt{x}}\,dx$$

   Solution: 2

   (g)
   $$\int_0^1 \frac{1}{x^{1/3}}\,dx$$

   Solution: 3/2

(h)

$$\int_0^1 \frac{1}{x^{1.2}}\,dx$$

Solution: Diverges

4. Determine the values of p for which the following integral converges:

$$\int_1^\infty \frac{1}{x^p}\,dx$$

Solution: $p > 1$

5. Determine the values of p for which the following integral converges:

$$\int_0^1 \frac{1}{x^p}\,dx$$

Solution: $0 < p < 1$