Warm-Up Problems and Lecture Problems
Solutions
February 12, 2003

1. Suppose you are given the following inequality:

   $$\frac{K(b-a)^5}{180n^4} < 0.1$$

   Suppose you know the following: $K = 10$, $a = 1$, $b = 5$.

   (a) What are some possible values for n that make the inequality hold?

   (b) What is the smallest value of n that will make this inequality hold?
   Solution: You can solve the inequality to get $n > 4.9$. We will see in class that we are interested in integers and often only even integers, so we will want $n \geq 6$.

2. Suppose you are given the following inequality:

   $$E \leq \frac{K(b-a)^5}{180n^4}$$

   Again, you have $K = 10$, $a = 1$ and $b = 5$ and you have control over n. You now want to ensure that $E < 0.001$.

   (a) What are some values of n what will force $E < 0.001$?

   (b) What is the smallest value of n that will force $E < 0.001$?
   Solution: Solving the inequality gives: $n > 15.4$ (or $n \geq 16$ if we insist that n be an integer).

   (c) Will all values of n larger then the value you found in the previous part ensure that $E < 0.001$? Why?

Lecture Problems

3. Given the error bound formula for Simpson's rule:

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$

Where K is some number such that $|f^{(4)}(x)| \leq K$ for all $a \leq x \leq b$. We want to investigate what this means for estimating:

$$\int_2^4 \ln x \, dx$$

(a) Find a suitable K.
Solution: Since $f(x) = \ln x$, $f^{(4)}(x) = -\frac{6}{x^4}$. Therefore, a suitable K would be $6/16 = 3/8$ (because for values of x between 2 and 4, $x = 2$ gives the minimum for $6/x^4$). (In fact, anything above 3/8 is also valid, but in some sense, 3/8 is the best choice for K.)

(b) Determine n so that the error in Simpson's rule is less then 0.00001.
Solution: Solve the inequality to get:

$$n^4 \geq \frac{K(b-a)^5}{(180)(0.00001)} = \frac{(3/8)2^5}{180(0.00001)} \cong 6667$$

Taking the 4-th root we get $n \geq 9$.

4. Consider the following integral (and function!):

$$G(t) = \int_1^t \frac{1}{x^3}\,dx$$

(a) Find a simple expression for $G(t)$ by using the Fundamental Theorem of Calculus (the evaluation part!)
Solution: Find an antiderivative and plug in and subtract:

$$G(t) = \frac{1}{2} - \frac{1}{2t^2}$$

(b) What happens to $G(t)$ as $t \to \infty$? (In other words, what is the limit: $\lim_{t\to\infty} G(t)$?)
Solution: 1/2.

(c) What does $t \to \infty$ correspond to in terms of area under the curve?