

Warm-Up Problems and Lecture Problems

Solutions February 10, 2003

1. Suppose you are given the following three points:

$$(-1, 1) \quad (0, 4) \quad (1, 3)$$

Your goal is to find the quadratic function that fits this data. In other words, you need to find A , B , and C so that for these three points:

$$y = Ax^2 + Bx + C$$

- (a) Find a system of three equations and three unknowns by plugging the data points into the equation.

Solution:

$$A - B + C = 1$$

$$C = 4$$

$$A + B + C = 3$$

- (b) Solve this system of equations and write down the quadratic function that fits the data.

Solution: Solving gives $A = -2$, $B = 1$ and $C = 4$ so the equation is:

$$y = -2x^2 + x + 4$$

2. Completing the square. In case you don't remember what this means, here is an example. We will complete the square for the expression $x^2 + 6x + 25$:

$$\begin{aligned}x^2 + 6x + 25 &= x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 25 \\&= (x^2 + 6x + 9) - 9 + 25 \\&= (x^2 + 6x + 9) + 16 \\&= (x + 3)^2 + 16\end{aligned}$$

Complete the square for the following expressions:

(a) $x^2 - 4x + 5$

Solution: $x^2 - 4x + 5 = (x - 2)^2 + 1$

(b) $5 - 4x - x^2$

Solution: $5 - 4x - x^2 = 9 - (x + 2)^2$

Lecture Problems

3. Using the table of integrals in the back of your book, evaluate the integral.

$$\int \frac{14x^3}{\sqrt{6+5x^2}} dx$$

Hint 1: You might have to do a preliminary substitution or algebraic manipulation.

Hint 2: You might want to use integral number 55:

$$\int \frac{u \, du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$$

Solution: Make the substitution $u = x^2$ to get (notice that $a = 6, b = 5$):

$$\begin{aligned} \int \frac{14x^3}{\sqrt{6+5x^2}} dx &= \int \frac{7u}{\sqrt{6+5u}} du \\ &= 7 \left(\frac{2}{75} \right) (5x^2 - 12) \sqrt{6+5x^2} + C \end{aligned}$$

4. Consider the integral:

$$\int_1^3 \frac{1}{x} dx$$

- (a) Using Simpson's Rule with $n = 6$, find an approximation for the definite integral:

Solution:

$$\begin{aligned} S_6 &= \frac{1}{9} \left(\frac{1}{1} + 4\frac{1}{(4/3)} + 2\frac{1}{(5/3)} + 4\frac{1}{(6/3)} + 2\frac{1}{(7/3)} + 4\frac{1}{(8/3)} + \frac{1}{(9/3)} \right) \\ &\cong 1.09894179894 \end{aligned}$$

- (b) Compute the integral using the Fundamental Theorem of Calculus.

Solution: $\ln 3 \cong 1.9861228867$

- (c) Compute the error of Simpson's rule (i.e., how far off is Simpson's Rule?).

Solution: $|E| = |S_6 - \ln 3| \cong 0.00032951027$.