

Warm-Up Problems and Lecture Problems

April 9, 2003

The warm up problems are for fun and to illustrate some ideas about adding up a series.

1. (This problem is supposed to be fun!) Consider the geometric series below:

$$3 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots$$

- (a) What is the first term a ?
- (b) What is the common ratio r ?
- (c) Does the series converge or diverge? If it converges, find the sum.

Here's another way to find this sum. Let S be the sum: $S = \sum_{n=0}^{\infty} 3(\frac{1}{10})^n$. Then $10S = 10 \sum_{n=0}^{\infty} 3(\frac{1}{10})^n$. Of course $10S - S = 9S$, which we compute below:

$$\begin{aligned} 10S &= 30 + 3 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots \\ S &= 3 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots \\ 10S - S = 9S &= 30 \end{aligned}$$

Therefore $S = \frac{30}{9} = \frac{10}{3}$.

2. Lets try this again, but this time with the series $S = \sum_{n=0}^{\infty} 3^n$. Let S denote the sum and we multiply by 3 and subtract:

$$\begin{aligned} 3S &= 3 + 9 + 27 + \cdots \\ S &= 1 + 3 + 9 + 27 + \cdots \\ 3S - S = 2S &= -1 \end{aligned}$$

Therefore $S = -\frac{1}{2}$.

Here's the question: Is there anything wrong with what we just did? Why or why not?

3. Here are a few similar problems for you to try (of course they all really diverge, but we're having fun).

(a) $S = \sum_{n=0}^{\infty} 3(2)^n$

(b) $S = \sum_{n=0}^{\infty} 7(-5)^n$

Lecture Problems

4. Determine which of the following terms describe the sequences:

convergent, divergent, conditionally convergent, absolutely convergent

(a) $\sum_{n=1}^{\infty} \frac{n}{2n+1}$

(b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2n+1}$

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{2n+1}$

(d) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(e) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$

(f) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(g) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{n}$