Warm-Up Problems and Lecture Problems
Solutions
April 9, 2003

The warm up problems are for fun and to illustrate some ideas about adding up a series.

1. (This problem is supposed to be fun!) Consider the geometric series below:

$$3 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots$$

   (a) What is the first term a?

   (b) What is the common ratio r?

   (c) Does the series converge or diverge? If it converges, find the sum.

   Here's another way to find this sum. Let S be the sum: $S = \sum_{n=0}^{\infty} 3(\frac{1}{10})^n$. Then $10S = 10\sum_{n=0}^{\infty} 3(\frac{1}{10})^n$. Of course $10S - S = 9S$, which we compute below:

$$\begin{aligned} 10S &= 30 + 3 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots \\ S &= 3 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots \\ 10S - S = 9S &= 30 \end{aligned}$$

   Therefore $S = \frac{30}{9} = \frac{10}{3}$.

2. Lets try this again, but this time with the series $S = \sum_{n=0}^{\infty} 3^n$. Let S denote the sum and we multipy by 3 and subtract:

$$\begin{aligned} 3S &= 3 + 9 + 27 + \cdots \\ S &= 1 + 3 + 9 + 27 + \cdots \\ 3S - S = 2S &= -1 \end{aligned}$$

   Therefore $S = \frac{-1}{2}$.
   Here's the question: Is there anything wrong with what we just did? Why or why not?

3. Here are a few similar problems for you to try (of course they all really diverge, but we're having fun).

   (a) $S = \sum_{n=0}^{\infty} 3(2)^n$
   Solution: $2S - S = -3$ so this sum is -3.

   (b) $S = \sum_{n=0}^{\infty} 7(-5)^n$
   Solution: $-5S - S = -6S = -7$, so this sum is $\frac{7}{6}$.

Lecture Problems

4. Determine which of the following terms describe the sequences:

   convergent, **divergent**, **conditionally convergent**, **absolutely convergent**

   (a) $\sum_{n=1}^{\infty} \frac{n}{2n+1}$
   Solution: Divergent!

   (b) $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n}{2n+1}$
   Solution: Divergent!

   (c) $\sum_{n=1}^{\infty}(-1)^{n} \frac{n}{2n+1}$
   Solution: Divergent!

   (d) $\sum_{n=1}^{\infty} \frac{1}{n^2}$
   Solution: Convergent, absolutely convergent.

   (e) $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{n^2}$
   Solution: Convergent, absolutely convergent. We already know it is absolutely convergent, so therefore this series is convergent.

   (f) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$
   Solution: Divergent! This is a comparison test $\frac{\ln n}{n} \geq \frac{1}{n}$.

   (g) $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\ln n}{n}$
   Solution: Convergent, conditionally convergent. All that needs to be checked is that $\lim \frac{\ln n}{n} = 0$ (which is a l'Hopitals rule) and that the series is decreasing. Just look at $f(x) = \frac{\ln x}{x}$ and compute $f'(x) = (1 - \ln x)/x^2$, which is always negative for $x > 1$. So, the series is decreasing.