

Warm-Up Problems and Lecture Problems

April 7, 2003

1. Consider the probability density function below:

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \leq x \leq 2 \\ 0 & x > 2 \end{cases}$$

- (a) Show that $f(x)$ is a probability density function.

- (b) Find the mean, μ , of the random variable.

- (c) Find the median of the random variable.

Hint: the median is the number m so that $\int_{-\infty}^m f(x) dx = 0.5$.

2. Suppose a sequence is given recursively as:

$$\begin{cases} a_1 = 1 \\ a_{n+1} = \frac{1}{4-a_n} \quad \text{if } n > 1 \end{cases}$$

Suppose we know that $\lim_{n \rightarrow \infty} a_n = L$, find L .

Lecture Problems

3. Lets find an estimate for $S = \sum_{n=1}^{\infty} \frac{1}{n^3}$.

(a) Using your calculator, find s_{10} .

(b) Find $\int_{10}^{\infty} \frac{1}{x^3} dx$.

(c) Find $\int_{11}^{\infty} \frac{1}{x^3} dx$.

(d) How does R_{10} relate to the previous integrals?

(e) What inequalities can we set up for the sum $S = \sum_{n=1}^{\infty} \frac{1}{n^3}$?

(f) What is a good estimate for the sum and at most, how far off can you be?