Warm-Up Problems and Lecture Problems
April 25, 2003

$$
\begin{aligned}
e^x &= 1 + x + \tfrac{x^2}{2!} + \tfrac{x^3}{3!} + \tfrac{x^4}{4!} + \cdots \\
\sin x &= x + \tfrac{x^3}{3!} + \tfrac{x^5}{5!} + \tfrac{x^7}{7!} + \cdots \\
\cos x &= 1 + \tfrac{x^2}{2!} + \tfrac{x^4}{4!} + \tfrac{x^6}{6!} + \cdots \\
\tfrac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + \cdots \\
\tan^{-1} x &= x - \tfrac{x^3}{3} + \tfrac{x^5}{5} - \tfrac{x^7}{7} + \cdots \\
\ln(1-x) &= -x - \tfrac{x^2}{2} - \tfrac{x^3}{3} - \tfrac{x^4}{4} - \cdots \\
\ln x &= (x-1) - \tfrac{(x-1)^2}{2} + \tfrac{(x-1)^3}{3} - \tfrac{(x-1)^4}{4} + \cdots
\end{aligned}
$$

1. Which Taylor polynomial should you use to approximate $\sin(\Pi/5)$ to within 0.001?
   Solution: Using the formula for the remainder (and notice that $M = 1$):

$$|R_n| \le \frac{|\pi/5|^{n+1}}{(n+1)!}$$

We now try plugging in various things for n:

n	$\frac{\lvert\pi/5\rvert^{n+1}}{(n+1)!}$
1	0.1973920881
2	0.04134170224
3	0.006493939406
4	0.0008160524934

So, we need to use T_3, but since $T_3 = T_4$ for $\sin x$, we will actually have accuracy to within 0.000816.

Lecture Problems

2. How accurate does the Taylor polynomial of degree 5, $T_5(x)$, approximate $\sin(\pi/5)$?
   Solution: : Using the formula for the remainder (and using the fact that $T_5 = T_6$):

$$|R_6| \le \frac{|\pi/5|^7}{7!} \approx 7.670585984^{-6}$$

3. For what values of x does the Taylor polynomial of degree 5 approximate $\sin(x)$ to within 0.001?
   Solution: Again, use the formula for the remainder and solve the inequality:

$$|R_6| \le \frac{|x|^7}{7!} \le 0.001$$

which gives

$$|x| \le 1.750673$$