Warm-Up Problems and Lecture Problems
April 2, 2003

Series Toolbox:

I. If a_n is a geometric sequence, then $\sum_{n=1}^{\infty} a_n$ converges if $|r| < 1$ and diverges if $|r| \geq 1$. If $|r| < 1$, then $\sum_{n=1}^{\infty} a_n = \frac{a}{1-r}$ where a is the first term and r is the common ratio.

II. If $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ diverges.

III. If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ might diverge or it might converge (thing about $\lim_{n\to\infty} \frac{1}{n}$).

1. Determine of the following series diverge or converge. If the series converges, try to find the sum.

   (a) $\sum_{n=1}^{\infty} 5^{-n}$

   (b) $\sum_{n=0}^{\infty} 5^{-n}$

   (c) $\sum_{n=1}^{\infty} \sqrt[n]{2}$

   (d) $\sum_{n=1}^{\infty} \frac{1+2^n}{5^n}$

   (e) $\sum_{n=0}^{\infty} \sin^n 1$

2. (a) Suppose you know that $\sum_{n=1}^{\infty} a_n$ converges. How about $\sum_{n=100}^{\infty} a_n$? Why or why not?

   (b) Suppose you know that $\sum_{n=1}^{\infty} a_n$ diverges. How about $\sum_{n=100}^{\infty} a_n$? Why or why not?

Lecture Problems

3. Use the integral test to determine if the following series converge or diverge

   (a) $\sum_{k=0}^{\infty} \frac{1}{k^2+1}$

   (b) $\sum_{n=1}^{\infty} \frac{4}{4n-1}$

4. Use the comparison test to determine if the following series converge or diverge

   (a) $\sum_{k=0}^{\infty} \frac{1}{k^2+1}$

   (b) $\sum_{n=1}^{\infty} \frac{4}{4n-1}$