Warm-Up Problems and Lecture Problems
Solutions
April 14, 2003

1. Find the domain of the following functions. (For which values of x does the series converge?) (Use the ratio test and the remember to test the "end points."

   (a) $f(x) = \sum_{n=0}^{\infty} x^n$
   Solution: $(-1, 1)$

   (b) $g(x) = \sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$
   Solution: $[2, 4)$

   (c) $h(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
   Solution: $(-\infty, \infty)$

   (d) $k(x) = \sum_{n=0}^{\infty} n! x^n$
   Solution: $x = 0$

   (e) $j(x) = \sum_{n=1}^{\infty} \frac{(x+2)^n}{2^n n^2}$
   Solution: $[-4, 0]$

Lecture Problems

2. For each of the power series below, determine where the series is centered, and then determine the radius of convergence and the interval of convergence.

Series	Center of Power Series	Radius of Convergence	Interval of Convergence
$\sum_{n=0}^{\infty} 3^n x^n$	$x = 0$	$R = \frac{1}{3}$	$\left(-\frac{1}{3}, \frac{1}{3}\right)$
$\sum_{n=0}^{\infty} 3^n (x-5)^n$	$x = 5$	$R = \frac{1}{3}$	$\left(5 - \frac{1}{3}, 5 + \frac{1}{3}\right)$
$\sum_{n=0}^{\infty} 3^n (2x-5)^n$	$x = \frac{5}{2}$	$R = \frac{1}{6}$	$\left(\frac{5}{2} - \frac{1}{6}, \frac{5}{2} + \frac{1}{6}\right)$
$\sum_{n=1}^{\infty} \frac{1}{n^3} (x+3)^n$	$x = -3$	$R = 1$	$[-1, 1]$
$\sum_{n=1}^{\infty} \frac{n}{(n+1)^2} (3x-4)^n$	$x = \frac{4}{3}$	$R = \frac{1}{3}$	$\left[\frac{4}{3} - \frac{1}{3}, \frac{4}{3} + \frac{1}{3}\right)$

3. Find a power series representation for the following functions:

   (a) $h(x) = \frac{x}{1-x}$.
   Solution: $h(x) = x \sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} x^{n+1} = \sum_{n=1}^{\infty} x^n$

   (b) $k(x) = \frac{x}{1-x^3}$
   Solution: $k(x) = x \sum_{n=0}^{\infty} (x^3)^n = \sum_{n=0}^{\infty} x^{3n+1}$

   (c) $g(x) = \frac{1}{2+x}$ (Hint: you can rearrange g to look like $g(x) = \frac{1}{2(1+x/2)}$
   Solution: $g(x) = \frac{1}{2} \sum_{n=0}^{\infty} (-x/2)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} x^n$