Name: _____
ID: _____

- 17 multiple choice questions worth 4.7 points each.
- 2 hand graded questions worth 10 points each.
- 0.1 "free" points (so the total will be 100).
- Exam covers sections 2.7 through 3.10

- No graphing calculators!
  Any non-graphing, non-differentiating, non-integrating scientific calculator is fine.
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. Your ability to make your solution clear will be part of the grade.

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$	$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\tan(A/2) = \dfrac{1 - \cos A}{\sin A} = \dfrac{\sin A}{1 + \cos A}$
$\sin^2(A/2) = \dfrac{1 - \cos A}{2}$	$\cos^2(A/2) = \dfrac{1 + \cos A}{2}$
$\sin A \sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$	$\cos A \cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$
$\sin A \cos B = \dfrac{1}{2}\left[\sin(A + B) + \cos(A - B)\right]$	
$\sin A + \sin B = 2 \sin\left(\dfrac{A + B}{2}\right) \cos\left(\dfrac{A - B}{2}\right)$	$\sin A - \sin B = 2 \cos\left(\dfrac{A + B}{2}\right) \sin\left(\dfrac{A - B}{2}\right)$
$\cos A + \cos B = 2 \cos\left(\dfrac{A + B}{2}\right) \cos\left(\dfrac{A - B}{2}\right)$	$\cos A - \cos B = -2 \sin\left(\dfrac{A + B}{2}\right) \sin\left(\dfrac{A - B}{2}\right)$
Law of Cos: $c^2 = a^2 + b^2 - 2ab \cos C$	Law of Sin: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$
$\dfrac{d}{dx}\left(\sin^{-1} x\right) = \dfrac{1}{\sqrt{1 - x^2}}$	$\dfrac{d}{dx}\left(\cos^{-1} x\right) = -\dfrac{1}{\sqrt{1 - x^2}}$
$\dfrac{d}{dx}\left(\tan^{-1} x\right) = \dfrac{1}{1 + x^2}$	

1. Let $f(x) = \dfrac{x^2}{x^3+1}$. Find $f'(1)$

   A. -1

   B. 0

   C. $1/16$

   D. $1/8$

   E. $1/4$

   F. $1/2$

   G. 1

   H. $3/2$

   I. 2

Solution: Quotient rule

$$\begin{aligned} f'(x) &= \frac{2x(x^3+1) - (x^2)(3x^2)}{(x^3+1)^2} \\ &= \frac{2x - x^4}{(x^3+1)^2} \\ f'(1) &= \frac{1}{4} \end{aligned}$$

2. Let $f(x) = \ln(x^2 - 3)$. Find $f'(2)$

   A. 0

   B. $\ln 2$

   C. 1

   D. 2

   E. e

   F. 3

   G. 4

   H. 5

Solution: Chain rule

$$f'(x) = \frac{2x}{x^2 - 3}$$
$$f'(2) = \frac{4}{1} = 4$$

3. Find $\frac{d}{dt}(t^5 + 3t^2 - e^4)^{2/3}$

A. $(t^5 + 3t^2 - e^4)^{2/3}$

B. $\frac{2}{3(t^5 + 3t^2 - e^4)^{1/3}}$

C. $\frac{5t^4 + 6t}{(t^5 + 3t^2 - e^4)^{1/3}}$

D. $\frac{5t^4 + 6t - 4e^3}{(t^5 + 3t^2 - e^4)^{1/3}}$

E. $\frac{2(5t^4 + 6t)}{3(t^5 + 3t^2 - e^4)^{1/3}}$

F. $\frac{\pi}{e^3}$

Solution: Chain rule

$$\frac{d}{dt}(t^5 + 3t^2 - e^4)^{2/3} = \frac{2}{3}(t^5 + 3t^2 - e^4)^{-1/3}(5t^4 + 6t)$$
$$= \frac{2(5t^4 + 6t)}{(t^5 + 3t^2 - e^4)^{1/3}}$$

4. Find $\frac{d}{dx}\sqrt{\tan x}$

A. $\frac{1}{2\sqrt{\sin x \cos^3 x}}$

B. $\frac{1}{\sqrt{\sin x \cos^3 x}}$

C. $\frac{2\cos^2 x}{\sqrt{\sin x}}$

D. $\frac{\sqrt{\cos^3 x}}{2\sqrt{\sin x}}$

E. $\frac{\sqrt{\sin x}}{\sqrt{\cos x}}$

F. $\sqrt{\sec^2 x}$

Solution:

$$\begin{aligned}\frac{d}{dx}\sqrt{\tan x} &= \frac{d}{dx}(\tan x)^{1/2}\\ &= \frac{1}{2}(\tan x)^{-1/2}(\tan x)' = \frac{1}{2}(\tan x)^{-1/2}(\sec^2 x)\\ &= \frac{(\cos x)^{1/2}}{2(\sin x)^{1/2}(\cos x)^2}\\ &= \frac{1}{2(\sin x)^{1/2}(\cos x)^{3/2}}\\ &= \frac{1}{2\sqrt{\sin x \cos^3 x}}\end{aligned}$$

5. Find $\frac{d}{dx}(e^{2x}\sin 3x)$

A. $e^{2x}(2\sin 3x + 3\cos 3x)$

B. $e^{2x}(3\sin 3x + 2\cos 3x)$

C. $e^{2x}(2\sin 3x - 3\cos 3x)$

D. $e^{2x}(3\sin 3x - 2\cos 3x)$

E. $2e^{2x}\sin 3x$

F. $3e^{2x}\cos 3x$

G. $5e^{2x}\cos 3x$

Solution:

$$\begin{aligned}(e^{2x}\sin 3x)' =&(e^{2x})'(\sin 3x) + (e^{2x})(\sin 3x)'\\ =&2e^{2x}(\sin 3x) + 3e^{2x}\cos 3x\\ =&e^{2x}(2\sin 3x + 3\cos 3x)\end{aligned}$$

6. Let $f(x) = e^{(x^3-2x^2)}$. Find $f'(2)$

   A. 0

   B. 1

   C. 2

   D. e

   E. 3

   F. 4

   G. e^2

Solution:

$$\begin{aligned}f'(x) =&(3x^2 - 4x)e^{x^3-2x^2}\\ f'(2) =&(12 - 8)e^0 = 4\end{aligned}$$

7. Let $y = mx + b$ be the tangent line to the graph of $f(x) = 3x^2 - 2$ at the point $(1, 1)$. What is $m + b$?

   A. -1

   B. 0

   C. 1

   D. 2

   E. 3

   F. 4

   G. 5

   H. 6

Solution: $f'(x) = 6x$, $f'(1) = 6$. Thus the equation of the tangent line is $y = 6x - 5$.

8. At the point $(1, 1)$, the parabola $y = ax^2 + bx$ has a tangent line equal to $y = 3x - 2$. Find $a - b$.

   A. -3

   B. -2

   C. -1

   D. 0

   E. 1

   F. 2

   G. 3

   H. $\pi/3$

Solution: Set up equations to solve for a and b. Since the point $(1, 1)$ is on the graph, we can plug this in and get the equation $1 = a + b$. We also know that slope of the tangent line at $x = 1$ is 3. Since $y' = 2ax + b$, we can plug in $y' = 3$ and $x = 1$ into this:

$$\begin{aligned} 1 =& a + b \\ 3 =& 2a + b \end{aligned}$$

Solving gives $a = 2$ and $b = -1$.

9. If $g(2) = 3$ and $g'(2) = -1$, find $\frac{d}{dx}\left(\frac{g(x)}{x}\right)$ at $x = 2$.

A. $-5/4$

B. -1

C. $-3/4$

D. $-1/4$

E. 0

F. $1/4$

G. $3/4$

H. 1

I. $5/4$

Solution:

$$\frac{d}{dx}\frac{g(x)}{x} = \frac{x\,g'(x) - g(x)}{x^2}$$

$$\left.\frac{d}{dx}\frac{g(x)}{x}\right|_{x=2} = \frac{2\,g'(2) - g(2)}{2^2} = \frac{2(-1) - (3)}{4} = -\frac{5}{4}$$

10. Find the value of c such that the line $y = \frac{3}{2}x + 6$ is tangent to the curve $y = c\sqrt{x}$.

A. It is not possible to find such a value of c.

B. 1

C. 2

D. 3

E. 6

F. 9

G. 25

H. 36

Solution: Taking the derivative: $y' = \frac{c}{2\sqrt{x}}$. We don't know the point that the tangency occurs, but lets say it is at $x = a$ and $y = b$. We have several conditions:

$$\begin{aligned} \frac{3}{2} &= \frac{c}{2\sqrt{a}} \qquad \text{(Because } y'(a) = 3/2\text{)} \\ b &= \frac{3}{2}a + 6 \\ b &= c\sqrt{a} \end{aligned}$$

We have three equations and three unknowns, we now solve (using your favorite method) to arrive at $a = 36/9$, $b = 12$, and $c = 6$.

11. Let f, g, h be differentiable functions. Here is a table of values for the functions and their derivatives.

$\mathbf{x}$	$\mathbf{f(x)}$	$\mathbf{f'(x)}$	$\mathbf{g(x)}$	$\mathbf{g'(x)}$	$\mathbf{h(x)}$	$\mathbf{h'(x)}$
-1	-2	2	-2	0	-2	0
0	3	3	-1	$1/2$	-1	1
1	0	-3	0	1	0	2
2	$1/2$	-1	$1/4$	2	2	1
3	$-1/2$	0	2	3	3	0

Find $(f \circ g \circ h)'(1)$.

A. -3

B. -2

C. -1

D. 0

E. 1

F. 2

G. 3

Solution:

$$\begin{aligned}(f \circ g \circ h)'(1) =& f'(g(h(1)))\ g'(h(1))\ h'(1)\\ =& f'(g(0))\ g'(0)\ (2)\\ =& f'(-1)\ (1/2)\ (2)\\ =& 2\ (1/2)\ (2) = 2\end{aligned}$$

12. If $g(x) + x\cos(g(x)) = x^2$, find $g'(0)$.

A. It is impossible to determine $g'(0)$.

B. 2

C. -1

D. $-1/2$

E. 0

F. 1

G. 2

Solution: Take the derivative (implicitly)

$$\begin{aligned}(g(x))' + (x\cos(g(x)))' =& (x^2)' \\ g'(x) + \cos(g(x)) - xg'(x)\sin(g(x)) =& 2x\end{aligned}$$

Plug in $x = 0$:

$$\begin{aligned}g'(0) + \cos(g(0)) + 0 =& 0 \\ g'(0) =& -\cos(g(0))\end{aligned}$$

We now need to find $g(0)$. We plug in $x = 0$ into the original equation

$$g(0) + 0 = 0$$

So $g(0) = 0$. Thus, $g'(0) = -\cos(0) = -1$.

13. If $f(x) = \ln(x + \ln x)$, find $f'(1)$.

A. 0

B. 1

C. 2

D. e

E. 3

F. e^2

G. 10

H. e^3

Solution:

$$f'(x) = \frac{1}{x + \ln x}(x + \ln x)' = \frac{1}{x + \ln x}\left(1 + \frac{1}{x}\right) = \frac{x+1}{x(x + \ln x)}$$

$$f'(1) = \frac{2}{1(1+0)} = 2$$

14. Find the equation of the tangent line to the curve $x^2 - xy - y^2 = 1$ at the point $(2, 1)$.

A. $y = 2x$

B. $y = \frac{1}{2}x + \frac{3}{4}$

C. $y = \frac{1}{2}x - \frac{3}{4}$

D. $y = -\frac{1}{2}x + \frac{3}{4}$

E. $y = \frac{3}{4}x + \frac{1}{2}$

F. $y = \frac{3}{4}x - \frac{1}{2}$

G. $y = -\frac{3}{4}x + \frac{1}{2}$

Solution: Take the derivative (implicitly):

$$\begin{aligned}(x^2)' - (xy)' - (y^2)' &= (1)' \\ 2x - (y + xy') - 2yy' &= 0 \\ y' &= \frac{2x - y}{x + 2y}\end{aligned}$$

Plug in $(x, y) = (2, 1)$ to get $y'(2, 1) = 3/4$. Thus the equation of the tangent line is $y = 1 + \frac{3}{4}(x - 2)$.

15. Let $L(x)$ be the linearization of $f(x) = \sqrt{x}$ at the point $x = 9$. Find $L(8)$.

A. $3 - \frac{1}{2}$

B. $3 - \frac{1}{3}$

C. $3 - \frac{1}{6}$

D. $3 - \frac{1}{9}$

E. $3 - \frac{1}{18}$

F. 3

G. $3 + \frac{1}{18}$

H. $3 + \frac{1}{9}$

I. $3 + \frac{1}{6}$

J. $3 + \frac{1}{3}$

K. $3 + \frac{1}{2}$

Solution: We have the formula for the tangent line: $L(x) = f(a) + f'(a)(x - a)$. Using $f(x) = \sqrt{x}$ and $a = 9$ gives.

$$L(x) = 3 + \frac{1}{6}(x - 9)$$

Plugging in $x = 8$ gives $L(8) = 3 - \frac{1}{6}$.

16. Find y'' by implicit differentiation when $\sin y - x = 1$

A. $\dfrac{1}{\cos y}$

B. $\dfrac{1}{\sin y - x}$

C. $\dfrac{1}{\sin y}$

D. $\dfrac{\sin y}{x+1}$

E. $\dfrac{\sin y}{\cos^2 y}$

F. $\dfrac{\sin^2 y}{\cos^2 y}$

G. $\dfrac{\sin y}{\cos^3 y}$

H. $\dfrac{\sin^2 y}{\cos^3 y}$

Solution: Take the derivative implicitly, twice

$$\begin{aligned}
(\sin y)' - (x)' &= (1)' \\
(\cos y)y' - 1 &= 0 \\
y' &= \sec y \\
y'' &= (\sec y \tan y)(y') \\
&= (\sec y \tan y)(\sec y) = \sec^2 y \tan y = \frac{\sin y}{\cos^3 y}
\end{aligned}$$

17. Suppose you have a function such that

$$f(x+h) - f(x) = h\cos h + h^2x^2 + 3hx$$

Find $f'(-1)$.

A. It is impossible to determine $f'(-1)$.

B. -2

C. -1

D. 0

E. 1

F. 2

G. 1001

H. h

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{h\cos h + h^2x^2 + 3hx}{h} \\ &= \lim_{h\to 0} \frac{h(\cos h + hx^2 + 3x)}{h} \\ &= \lim_{h\to 0} \cos h + hx^2 + 3x = 1 + 3x \end{aligned}$$

Thus, $f'(-1) = -2$.

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Written Problem. You will be graded on the readability of your work.
Use the back of this sheet, if necessary.

18. Use the limit definition of derivative to find the derivative of

$$f(x) = \frac{1}{\sqrt{x+1}}$$

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h+1}} - \frac{1}{\sqrt{x+1}}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{x+1}-\sqrt{x+h+1}}{h\sqrt{x+h+1}\sqrt{x+1}} \\
&= \lim_{h\to 0} \left(\frac{\sqrt{x+1}-\sqrt{x+h+1}}{h\sqrt{x+h+1}\sqrt{x+1}}\right) \cdot \left(\frac{\sqrt{x+1}+\sqrt{x+h+1}}{\sqrt{x+1}+\sqrt{x+h+1}}\right) \\
&= \lim_{h\to 0} \frac{(x+1)-(x+h+1)}{h\sqrt{x+h+1}\sqrt{x+1}(\sqrt{x+1}+\sqrt{x+h+1})} \\
&= \lim_{h\to 0} \frac{-h}{h\sqrt{x+h+1}\sqrt{x+1}(\sqrt{x+1}+\sqrt{x+h+1})} \\
&= \lim_{h\to 0} \frac{-1}{\sqrt{x+h+1}\sqrt{x+1}(\sqrt{x+1}+\sqrt{x+h+1})} \\
&= \frac{-1}{\sqrt{x+1}\sqrt{x+1}(\sqrt{x+1}+\sqrt{x+1})} \\
&= \frac{-1}{(x+1)(2\sqrt{x+1})} \\
&= \frac{-1}{2(x+1)^{3/2}} = -\frac{1}{2}(x+1)^{3/2}
\end{aligned}$$

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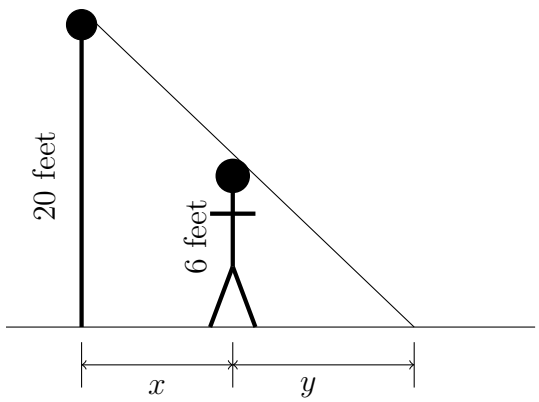
Written Problem. You will be graded on the readability of your work.
Use the back of this sheet, if necessary.

19. On a dark night, a 6 foot tall person is walking away from a lamp post at a rate of 10 feet per second. The lamp post is 20 feet high. When the person is 15 feet from the lamp post, at what rate is the person's shadow growing or shrinking?

Solution: Start with a picture.



We know $\frac{dx}{dt} = 10$. We want to find $\frac{dy}{dt}$ when $x = 15$. We need to relate x and y. To do this, we use similar triangles:

$$\frac{20}{x+y} = \frac{6}{y}$$

We simplify this to get the equation: $10y = 3x + 3y$ or, even better, $7y = 3x$. We now take derivatives and get $7\frac{dy}{dt} = 3\frac{dx}{dt}$. Plugging in $\frac{dx}{dt} = 10$ gives $\frac{dy}{dt} = \frac{30}{7}$.