Name: ________________________________
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- 17 multiple choice questions worth 4 points each.
- 2 hand graded questions worth 16 points each.
- Exam covers sections 1.1 through 2.6.

- No graphing calculators!
  Any non-graphing, non-differentiating, non-integrating scientific calculator is fine.
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. Your ability to make your solution clear will be part of the grade.

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$	$\sin(2A) = 2\sin A \cos A$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\cos(2A) = \cos^2 A - \sin^2 A$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\tan(2A) = \dfrac{2\tan A}{1 - \tan^2 A}$
$\sin^2(A/2) = \dfrac{1 - \cos A}{2}$	$\cos^2(A/2) = \dfrac{1 + \cos A}{2}$
$\tan(A/2) = \dfrac{1 - \cos A}{\sin A} = \dfrac{\sin A}{1 + \cos A}$	
$\sin A \sin B = \dfrac{1}{2}[\cos(A - B) - \cos(A + B)]$	$\cos A \cos B = \dfrac{1}{2}[\cos(A - B) + \cos(A + B)]$
$\sin A \cos B = \dfrac{1}{2}[\sin(A + B) + \cos(A - B)]$	$\cos A \sin B = \dfrac{1}{2}[\sin(A + B) - \cos(A - B)]$
$\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$	$\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$
$\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$	$\cos A - \cos B = -2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$
Law of Cos: $c^2 = a^2 + b^2 - 2ab\cos C$	Law of Sin: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

1. Find the limit

$$\lim_{x\to 0} \frac{\cos x}{x-2}$$

A. 1
B. 0
C. $\frac{1}{2}$
D. 2
E. -1
F. $-\frac{1}{2}$
G. -2
H. Does not exist

Solution: This is one where plugging in works:

$$\lim_{x\to 0} \frac{\cos x}{x-2} = \frac{\cos 0}{0-2} = -\frac{1}{2}$$

2. Find the domain of the function

$$f(x) = \frac{\sqrt{x}}{1-\frac{1}{1+x}}$$

A. $(-\infty, \infty)$
B. $(-\infty, 0)$
C. $(-\infty, 0]$
D. $[0, \infty)$
E. $(0, \infty)$
F. $(-\infty, -1) \cup (-1, \infty)$
G. $(-\infty, 1) \cup (1, \infty)$
H. $(-\infty, -1) \cup (1, \infty)$

Solution: We have to make sure that we don't square root a negative so we need $x \geq 0$. We also need to make sure the denominator(s) are not negative. So we can't have $x = -1$ and we can't have $1 - \frac{1}{1+x} = 0$. Solving this gives $x = 0$. Putting all these together give the domain of $(0, \infty)$

3. Let $f(x) = 2 \cdot 5^{4x}$. Write it in the form $y = Ae^{kx}$.

A. $y = 2 \cdot 5^{4x}$

B. $y = 2 \cdot 4^{x}$

C. $y = 2 \cdot e^{4x}$

D. $y = 2 \cdot e^{5x}$

E. $y = 2 \cdot e^{(5\ln 4)x}$

F. $y = 2 \cdot e^{(4\ln 5)x}$

G. $y = e^{20x}$

H. $y = e^{40x}$

Solution:

$$
\begin{aligned}
y =& 2 \cdot 5^{4x} = 2 \cdot e^{\ln 5^{4x}} = 2 \cdot e^{4x \ln 5} \\
A =& 2 \\
k =& 4 \ln 5
\end{aligned}
$$

4. Let $f(x) = x^2$ and $g(x) = \dfrac{4}{5+x}$. Find $(fg - f \circ g)(-3)$

   A. -3

   B. 0

   C. 3

   D. 10

   E. 14

   F. 20

   G. 100

   H. Does not exist

Solution:

$$\begin{aligned}
(fg - f \circ g)(x) =& x^2 \cdot \frac{4}{5+x} - f\left(\frac{4}{5+x}\right) \\
=& \frac{4x^2}{5+x} - \left(\frac{4}{5+x}\right)^2 \\
=& \frac{4x^2}{5+x} - \frac{16}{(5+x)^2} \\
(fg - f \circ g)(-3) =& \frac{4(-3)^2}{5+(-3)} - \frac{16}{(5+(-3))^2} \\
=& \frac{36}{2} - \frac{16}{4} = 18 - 4 = 14
\end{aligned}$$

5. Find the inverse of the function $f(x) = \frac{1}{3}\sqrt{x-5} + 2$

   A. $f^{-1}(x) = \frac{1}{3}\sqrt{x-5} + 2$

   B. $f^{-1}(x) = \frac{3}{\sqrt{x-5}} + \frac{1}{2}$

   C. $f^{-1}(x) = \frac{1}{\frac{1}{3}\sqrt{x-5}+2}$

   D. $f^{-1}(x) = x^2 - 4x + 4$

   E. $f^{-1}(x) = x^2 - 4x + 9$

   F. $f^{-1}(x) = 9x^2 - 36x + 36$

G. $f^{-1}(x) = 9x^2 - 36x + 41$

H. The function $f(x)$ does not have an inverse.

Solution:

$$\begin{aligned} y =& \frac{1}{3}\sqrt{x-5} + 2 \\ 3(y-2) =& \sqrt{x-5} \\ 9(y-2)^2 =& x - 5 \\ 9(y-2)^2 + 5 =& x \\ x =& 9y^2 - 36y + 41 \\ f^{-1}(y) =& 9y^2 - 36y + 41 \\ f^{-1}(x) =& 9x^2 - 36x + 41 \end{aligned}$$

6. Solve for x:

$$\ln(\ln x) = 1$$

A. $x = 0$

B. $x = 1$

C. $x = e$

D. $x = e^2$

E. $x = e^e$

F. $x = \ln 2$

G. $x = \ln 4$

Solution:

$$\begin{aligned} \ln(\ln x) &= 1 \\ e^{\ln(\ln x)} &= e^1 = e \\ \ln x &= e \\ e^{\ln x} &= e^e \\ x &= e^e \end{aligned}$$

7. If $f(x) = \frac{1}{2} + \frac{e^{(x-4)}}{2}$. What is $f^{-1}(1)$?

A. $x = -1$

B. $x = 0$

C. $x = 1$

D. $x = 2$

E. $x = 3$

F. $x = 4$

G. $x = 5$

H. $x = \pi$

Solution: You need to solve $f(x) = 1$:

$$\begin{aligned}
\frac{1}{2} + \frac{1}{2}e^{(x-4)} &= 1 \\
1 + e^{(x-4)} &= 2 \\
e^{(x-4)} &= 1 \\
\ln(e^{(x-4)}) &= \ln(1) = 0 \\
x - 4 &= 0 \\
x &= 4
\end{aligned}$$

8. Find the limit

$$\lim_{x\to 2} \frac{x^2-4}{x^2-x-2}$$

A. 0

B. $\frac{1}{2}$

C. $\frac{3}{4}$

D. 1

E. $\frac{5}{4}$

F. $\frac{4}{3}$

G. 2

H. $-\infty$

I. ∞

J. Does not exist

Solution: If you plug in $x = 2$ you get $\frac{0}{0}$ so you have to try something algebraic:

$$\begin{aligned}\lim_{x\to 2} \frac{x^2-4}{x^2-x-2} &= \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)(x+1)} \\ &= \lim_{x\to 2} \frac{x+2}{x+1} = \frac{4}{3}\end{aligned}$$

9. Find the limit

$$\lim_{x\to 0} \frac{x}{\sqrt{7}-\sqrt{7+x}}$$

A. $-2\sqrt{7}$

B. $-\sqrt{7}$

C. 0

D. $\sqrt{7}$

E. $2\sqrt{7}$

F. Does not exist

Solution: Rationalize the denominator:

$$\begin{aligned}
\lim_{x\to 0} \frac{x}{\sqrt{7}-\sqrt{7+x}} &= \lim_{x\to 0} \frac{x}{\sqrt{7}-\sqrt{7+x}} \cdot \frac{\sqrt{7}+\sqrt{7+x}}{\sqrt{7}+\sqrt{7+x}} \\
&= \lim_{x\to 0} \frac{x(\sqrt{7}+\sqrt{7+x})}{7-(7+x)} \\
&= \lim_{x\to 0} \frac{x(\sqrt{7}+\sqrt{7+x})}{-x} \\
&= \lim_{x\to 0} -(\sqrt{7}+\sqrt{7+x}) = -2\sqrt{7}
\end{aligned}$$

10. Find the domain of $f(x) = (\sqrt{2-x})(\ln x)$

A. $(-\infty, \infty)$

B. $(0, \infty)$

C. $(-\infty, 2)$

D. $(0, 2)$

E. $(0, 2]$

F. $(2, \infty)$

G. $[2, 4]$

Solution: Because of the square root, we must have $2 - x \geq 0$, or $x \leq 2$. Because of the $\ln x$, we must have $x > 0$. Thus the domain is $(0, 2]$.

11. Find the limit

$$\lim_{h \to 1^-} \frac{|h-1|}{h-1}$$

A. -1

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. 1

F. Does not exist

Solution: Note that

$$|h-1| = \begin{cases} h-1 \text{ if } h-1 \geq 0 \\ -(h-1) \text{ if } h-1 < 0 \end{cases}$$

Thus, if we are approaching 1 from the left, $h - 1 < 0$ and $|h - 1| = -(h - 1)$. So:

$$\begin{aligned} \lim_{h \to 1^{-1}} \frac{|h-1|}{h-1} &= \lim_{h \to 1^{-1}} \frac{-(h-1)}{h-1} \\ &= \lim_{h \to 1^{-1}} -1 = -1 \end{aligned}$$

12. Suppose you know the following about the functions $f(x)$ and $g(x)$.

$$\lim_{x\to 3} f(x) = 4 \qquad \lim_{x\to 3} g(x) = 5$$
$$f(3) = 9 \qquad g(3) = 3$$

Find

$$\lim_{x\to 3} \frac{\sqrt{f(x)} + g(x)}{1 - (g(x))^2}$$

A. -1

B. $-\frac{7}{8}$

C. $-\frac{6}{8}$

D. $-\frac{5}{8}$

E. $-\frac{1}{3}$

F. $-\frac{7}{24}$

G. $-\frac{1}{4}$

H. $-\frac{5}{24}$

I. 0

J. 1

K. Does not exist

Solution:

$$\lim_{x\to 3} \frac{\sqrt{f(x)} + g(x)}{1 - (g(x))^2} = \frac{\sqrt{\lim_{x\to 3} f(x)} + \lim_{x\to 3} g(x)}{1 - (\lim_{x\to 3} g(x))^2}$$
$$= \frac{\sqrt{4} + 5}{1 - (5)^2} = -\frac{7}{24}$$

13. Let

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x < -1 \\ 1 & \text{if } x = -1 \\ (x+3)^3 - 5 & \text{if } x > -1 \end{cases}$$

Find $\lim_{x\to -1} f(x)$

A. -5

B. -3

C. -2

D. -1

E. 0

F. 1

G. 2

H. 3

I. 4

J. 5

K. Does not exist

Solution: You will need to check the left and right hand limits and see if they exist and are the same. The function value at 1 is irrelevant.

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1} x^2 + 1 = 3$$
$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1} (x+3)^3 - 5 = 3$$

Thus $\lim_{x\to -1} f(x) = 3$.

14. Find the constant c that makes the function g continuous on $(-\infty, +\infty)$.

$$g(x) = \begin{cases} x^2 & \text{if } x < 2 \\ cx + 10 & \text{if } x \geq 2 \end{cases}$$

A. $c = -3$

B. $c = -2$

C. $c = -1$

D. $c = 0$

E. $c = 1$

F. $c = 2$

G. $c = 3$

H. $c = 4$

I. $c = 10$

J. It is not possible to find such a c.

Solution: The problem point is at $x = 2$. The function is continuous at all other x values. We have to make sure:

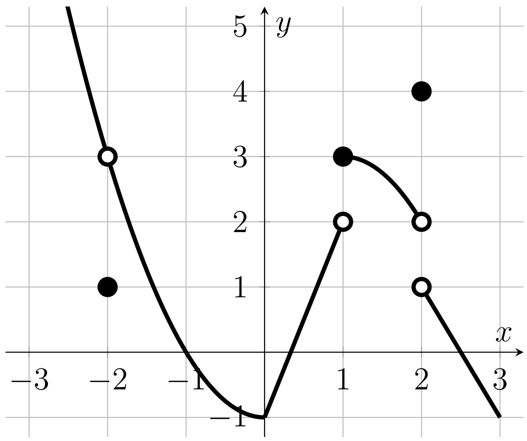
$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

So we compute all these:

$$\begin{aligned} \lim_{x \to 2^-} f(x) &= \lim_{x \to 2} x^2 = 4 \\ \lim_{x \to 2^+} f(x) &= \lim_{x \to 2} cx + 10 = 2c + 10 \\ f(2) &= 2c + 10 \end{aligned}$$

We need these to all be equal so we just solve $2c + 10 = 4$ and we get $c = -3$.

15. Consider the graph of the function $F(x)$ below.



Find

$$\left(\lim_{x\to -2} F(x)\right)+F(-2)+\left(\lim_{x\to 1^-} F(x)\right)+\left(\lim_{x\to 1^+} F(x)\right)+\left(\lim_{x\to 2^-} F(x)\right)+\left(\lim_{x\to 2^+} F(x)\right)+F(2)$$

A. $-\pi$ B. 0 C. 12 D. 13 E. 14 F. 15 **G.** 16 H. 17 I. 18
J. ∞ K. At least one of these does not exist so we can't find a sum

Solution:

$$\begin{aligned}
\lim_{x\to -2} F(x) &= 3\\
F(-2) &= 1\\
\lim_{x\to 1^-} F(x) &= 2\\
\lim_{x\to 1^+} F(x) &= 3\\
\lim_{x\to 2^-} F(x) &= 2\\
\lim_{x\to 2^+} F(x) &= 1\\
F(2) &= 4
\end{aligned}$$

Summing these gives the result of 16.

16. Which of the following functions below, all defined on the real line, have an inverse function?

    I. $h(x) = -5(x+5)^2 + 45$
    II. $g(x) = 1 + e^{4x}$
    III. $f(x) = \sin x$

    A. None of these
    B. I only
    C. II only
    D. III only
    E. I and II only
    F. I and III only
    G. II and III only
    H. I, II and III

Solution:

I. This function is not one to one and can not have an inverse without restricting the domain.

II. This function is one to one and has an inverse.

III. This function is not one to one and can not have an inverse without restricting the domain.

17. Compute

$$\lim_{x\to\infty} \frac{8x^4 - 12x + 1023}{4x^4 - 1228x^2 + 654x + 15}$$

A. $-\infty$
B. 0
C. 2
D. 4

E. 8

F. ∞

G. Does not exist

Solution: Multiply numerator and denominator by x^4 and take the limit of the terms indivually.

$$\begin{aligned}
\lim_{x\to\infty} \frac{8x^4 - 12x + 1023}{4x^3 - 1228x^2 + 654x + 15} &= \lim_{x\to\infty} \frac{\frac{8x^4}{x^4} - \frac{12x}{x^4} + \frac{1023}{x^4}}{\frac{4x^3}{x^4} - \frac{1228x^2}{x^4} + \frac{654x}{x^4} + \frac{15}{x^4}} \\
&= \lim_{x\to\infty} \frac{8 - \frac{12}{x^3} + \frac{1023}{x^4}}{4 - \frac{1228}{x^2} + \frac{654}{x^3} + \frac{15}{x^4}} \\
&= \frac{8 - 0 + 0}{4 - 0 + 0 + 0} = \frac{8}{4} = 2
\end{aligned}$$

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ID: ____________________

Written Problem. You will be graded on the readability of your work.
Use the back of this sheet, if necessary.

18.

$$f(x) = \begin{cases} 3 & \text{if } x < -1 \\ 2x & \text{if } -1 \leq x \leq 1 \\ \frac{6x+2}{x+3} & \text{if } x > 1 \end{cases}$$

Find the following:

(a) $\lim_{x \to -1^-} f(x)$

(b) $\lim_{x \to -1^+} f(x)$

(c) $f(-1)$

(d) $\lim_{x \to 1^-} f(x)$

(e) $\lim_{x \to 1^+} f(x)$

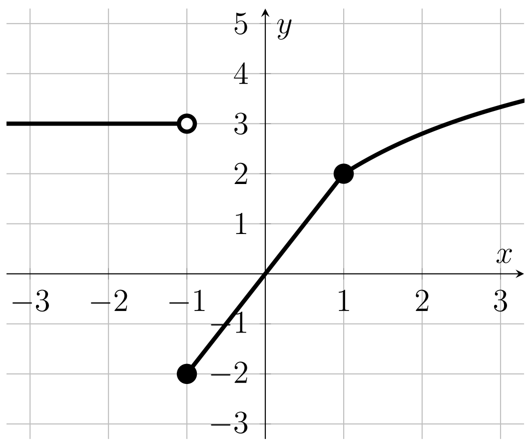
(f) $f(1)$

(g) $\lim_{x \to -\infty} f(x)$

(h) $\lim_{x \to \infty} f(x)$

(i) Describe all points where $f(x)$ is continuous and all points where $f(x)$ is not continuous. Explain.

Solution: Here's the graph of $f(x)$.



(a) $\lim_{x \to -1^-} f(x) = 3$

(b) $\lim_{x \to -1^+} f(x) = -2$

(c) $f(-1) = -2$

(d) $\lim_{x \to 1^-} f(x) = 2$

(e) $\lim_{x \to 1^+} f(x) = 2$

(f) $f(1) = 2$

(g) $\lim_{x \to -\infty} f(x) = -3$

(h) $\lim_{x \to \infty} f(x) = 5$

(i) f is continuous everywhere except $x = -1$. At this point, $\lim_{x \to -1} f(x)$ does not exist.

Name: ______________________________
ID: ______________________________

Written Problem. You will be graded on the readability of your work.
Use the back of this sheet, if necessary.

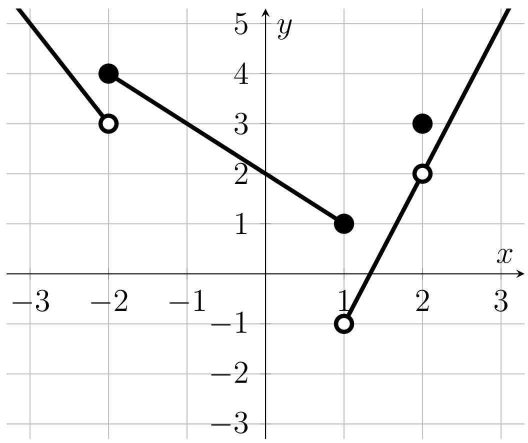
19. Suppose the function $f(x)$ satisfies the following:

$$\lim_{x\to -2^-} f(x) = 3 \qquad \lim_{x\to -2^+} f(x) = 4$$
$$\lim_{x\to 1^-} f(x) = 1 \qquad \lim_{x\to 1^+} f(x) = -1$$
$$\lim_{x\to 2} f(x) = 2 \qquad f(2) = 3$$

(a) Draw a possible graph of a function $f(x)$ satisfying these properties.

(b) Describe all points where function $f(x)$ is continuous and where $f(x)$ is not continuous? Explain.

Solution: Here is one possible answer.



This function is continuous at all values except $x = -2, 1, 2$.

At $x = -2$, $\lim_{x \to -2} f(x)$ does not exist.

At $x = 1$, $\lim_{x \to 1} f(x)$ does not exist.

At $x = 2$, $\lim_{x \to 2} f(x)$ exists, but $\lim_{x \to 2} f(x) \neq f(2)$.