Math 131 - April 29, 2016
Solutions

Final's Week Resources

- Blake's office hours (TBA)
- Marie Jose's office hours (TBA)
- Calculus help room
- RPMs

Test Review Problems

1. True/False

   (a) The function $f(x) = e^x$ is the inverse of $g(x) = \ln x$ **Solution:** True (SP2001, Final)

   (b) $\lim_{x\to 0}(\sin x)/x = 0$ **Solution:** False (SP2001, Final)

   (c) $f(x) = |x|$ is continuous for all x. **Solution:** True (SP2001, Final)

   (d) A function can have two different horizontal asymptotes. **Solution:** True (FL2001, Final)

   (e) If $-x^2 \leq f(x) \leq x^4$ for all x in $[-1, 1]$ then $\lim_{x\to 0} f(x) = 0$ **Solution:** True (FL2001, Final)

   (f) If $f''(c) = 0$ then $x = c$ is an inflection point. **Solution:** False (FL2001, Final)

   (g) If $f(x)$ is continuous then it is differentiable. **Solution:** False (FL2001, Final)

   (h) If $f(x)$ is differentiable then it is continuous. **Solution:** True

   (i) f and g are continuous on $[a, b]$ and $f(x) \geq g(x) \geq 0$, $m \leq f(x) \leq M$. Which are true:

   - $\int_a^b f(x)\, dx \geq 0$ **Solution:** True
   - $\int_a^b f(x)\, dx \geq \int_a^b g(x)\, dx$ **Solution:** True
   - $m(b - a) \leq \int_a^b f(x)\, dx \leq M(b - a)$ **Solution:** True

   (SP2005, Final)

2. Find the maximum and minimum of $f(x) = x^3 - 6x^2 + 9x + 1$ on $[2, 4]$.
   Solution: 1, 5

3. Find $\frac{d}{dx}\sin^2(x^2)$
   Solution: $4x \sin x^2 \cos x^2$

4. What is the slope of the line tangent to $x^2 + 3x^2y^2 + 5y^3 + y = 8$ at the point $(2, 0)$?
   Solution: -12

5. Find the equation of the line tangent to the curve $y = x^2 + e^x$ at the point $(1, 1 + e)$.
   Solution: $y = (2 + e)x - 1$ (SP2001, Final)

6. A particle's position at time t is $s(t) = \sin 2t$. What is the acceleration at time t?
   Solution: $-4 \sin 2t$ (SP2001, Final)

7. Find the average rate of change of $f(x) = x + 6/x$ on the interval $[1, 3]$.
Solution: -1 (SP2001, Final)

8. Find where $f(x) = 12/(x^2 + 6x + 12)$ is increasing.
Solution: $x < -3$ (SP2001, Final)

9. Find $\lim_{x\to 0}(e^x - 1 - x)/(\cos x - 1)$
Solution: -1 (SP2001, Final)

10. Find the minimum of $f(x) = (x^4 + 1/x)e^{-x}$ on the interval $(0, \infty)$.
Solution: No min. This is a tricky one and requires more subtle techniques than our usual routine of taking the derivative, setting it equal to zero, etc. The key points are the following:

    - $f(x) \geq 0$ for all $x > 0$
    - $\lim_{x\to 0^+} f(x) = +\infty$
    - $\lim_{x\to infty} f(x) = 0$

    Thus, there can be no global minimum. (Why? Try to see what these facts mean the graph looks like.) (SP2001, Final)

11. Use a linear approximation of $f(x) = x^{4/3}$ at $a = 1000$ to approximate $(1006)^{4/3}$.
Solution: 10080 (SP2001, Final)

12. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the bae of the triangle changing when the altitude is 10 cm and the area is 100 cm^2.
Solution: -1.6 cm/min (SP2001, Final)

13. Find the anti-derivative of $f(x) = x^3 + \sin x$
Solution: $\frac{1}{4}x^4 - \cos x - 4$ (SP2001, Final)

14. Find the minimum of $f(x) = x^2 \ln x$ on $[0, 2]$.
Solution: $1/\sqrt{e}$ (SP2001, Final)

15. Let $f(x) = (\ln(1 + 2x))/\cos 3x$. Find $f'(0)$
Solution: 2 (SP2001, Final)

16. A particle moves on a vertical line so that its y-coordinate at time t is $y = t^3 - 12t + 3$, for $t \geq 0$. When is the particle moving up?
Solution: $t > 2$ (FL2001, Final)

17. Find the equation of the tangent line to the curve $x^3 + y^3 = 6xy$ at the point $(3, 3)$.
Solution: $x + y = 6$ (FL2001, Final)

18. Find the equation of the tangent line to the curve $y = \sqrt{\sin^2 x + x^2}$ at (π, π)
Solution: $y = x$ (FL2001, Final)

19. Let $f(x) = e^x/\sin x$. Find where $f'(x) = 0$.
Solution: At $x = \pi/4$ (FL2001, Final)

20. Find $\lim_{x\to\infty} \sqrt{3x^2 + 1}/(2x - 4)$
Solution: $\sqrt{3}/2$ (FL2001, Final)

21. Suppose $f'(x) = e^x \sin x$. On the interval $(0, 2\pi)$, where is $f(x)$ concave down?
Solution: $3\pi/4 < x < 7\pi/4$ (FL2001, Final)

22. Find $\lim_{x\to 1} x^{1/(1-x)}$
Solution: $1/e$ (FL2001, Final)

23. What are all the indeterminate forms?
Solution: $0/0$, ∞/∞, 0^0, $\infty - \infty$, ∞^0, $0 \cdot \infty$, 0^1. (FL2001, Final)

24. Find the inflection points of $f(x) = e^{-2x^2}$.
Solution: $x = -1/2$ and $x = 1/2$ (FL2001, Final)

25. $f(x) = x^{1/x}$. Find $f'(2)$.
Solution: $\sqrt{2}/4(1 - \ln 2)$ (FL2001, Final)

26. Find all asymptotes of $f(x) = \frac{x^2-5x+6}{3x^2+4x+1}$
Solution: Horz at $y = 1/3$. Vert at $x = -1/3$ and $x = -1$. (FL2001, Final)

27. A spotlight on the ground shines on a wall 12 meters away. If a man 2 meters tall walks away from the spotlight towards the building at the rate of 1.6 m/s, how fast is the length of his shawdow decreasing when he is 4 m from the building.
Solution: 0.6 m/s (FL2001, Final)

28. Find $\lim_{x\to\infty} \ln((x+1)/(3x))$
Solution: $\ln(1/3)$ (SP2002, Final)

29. Find the inverse of $f(x) = (x+1)/(2x+1)$
Solution: $(1-x)/(2x-1)$ (FL2002, Final)

30. Use Riemann sums to compute $\int_0^4 (x+1)\, dx$. Check your solution with the fundamental theorem of calculus.
Solution: 12 (SP2004, Final)

31. If $\int_0^7 g(x)\, dx = 20$, $\int_4^7 g(x)\, dx = 4$, and $\int_0^1 g(x)\, dx = 7$ then what is $\int_1^4 g(x)\, dx$.
Solution: 9 (FL2004, Final)

32. Find $\int_0^{\pi/4} \sec^2 x + \cos x\, dx$
Solution: $(2 + \sqrt{2})/2$ (FL2004, Final)

33. Approximate $\int_{-4}^4 x^2 - 2x\, dx$ using a right hand sum with $n = 4$.
Solution: 32 (FL2005, Final)

34. Given theat $f(x) = \int_1^x \frac{1}{t}\, dt$, find $f'(2)$
Solution: 1/2 (FL2005, Final)

35. Let $F(x)$ be the antiderivative of $f(x) = 5x^4 - 2x^5$. Suppose $F(0) = 4$. Find $F(1)$.
Solution: 14/3 (SP2006, Final)

36. Find $\sum_{k=0}^5 k$
Solution: 15 (FL2004, Final)

37. Find $\int_1^2 (6x^2 - 4x + 5)\, dx$
Solution: 13 (FL2004, Final)

38. Write the integral as a limit of Riemann sums: $\int_0^4 (2x^2+3)\,dx$
Solution: $\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 \cdot \left(\frac{4i}{n}\right)^2 + 3\right) \left(\frac{4}{n}\right)$

39. Write the integral as a limit of Riemann sums: $\int_1^5 (2x^2+3)\,dx$
Solution: $\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 \cdot \left(1+\frac{4i}{n}\right)^2 + 3\right) \left(\frac{4}{n}\right)$

40. Write the limit as a definite integral: $\lim_{n\to\infty} \sum_{i=1}^{n} \left(-1+\frac{3i}{n}\right)^2 \left(\frac{3}{n}\right)$
Solution: $\int_0^3 (-1+x)^2\,dx$