Math 131 - April 20, 2016
Solutions

Warm-up Problems

1. Compute the following sums

   (a) $\sum_{i=1}^{10}(2i-3) = 80$

   (b) $\sum_{i=3}^{8}(2i^3 - i^2 + 2) = 2387$

Lecture Problems

2. Using geometry, compute $\int_0^4 2x + 1\, dx = 20$

3. Using limits of Riemann Sums, compute $\int_0^4 2x + 1\, dx$

   (a) $\Delta x = \frac{4}{n}$

   (b) $x_i = 0 + i\Delta x = \frac{4i}{n}$

   (c) Continue with this and simplify (using what you found above)

$$\begin{aligned}
\text{RHS} &= \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} f(4i/n)\cdot\frac{4}{n} \\
&= \sum_{i=1}^{n}\left(2\cdot\frac{4i}{n}+1\right)\cdot\frac{4}{n} \\
&= \sum_{i=1}^{n}\left(\frac{32i}{n^2}+\frac{4}{n}\right) \\
&= \frac{32}{n^2}\sum_{i=1}^{n} i + \frac{4}{n}\sum_{i=1}^{n} 1 \\
&= \frac{32}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{4}{n}(n) \\
&= \frac{32n(n+1)}{2n^2} + \frac{4n}{n}
\end{aligned}$$

   (d) Take the limit as $n \to \infty$

$$\begin{aligned}
\lim_{n\to\infty}\text{RHS} &= \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\text{(What you got in last problem)} \\
&= \lim_{n\to\infty}\left(\frac{32n(n+1)}{2n^2}+\frac{4n}{n}\right) = 16+4 = 20
\end{aligned}$$

4. Compute $\int_1^4 x^2\,dx$

(a) $\Delta x = \frac{3}{n}$

(b) $x_i = 1 + \frac{3i}{n}$

(c) Continue with this and simplify (using what you found above)

$$\begin{aligned}
\text{RHS} &= \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} f(1+3i/n)\cdot\frac{3}{n} \\
&= \sum_{i=1}^{n} (1+3i/n)^2\cdot\frac{3}{n} \\
&= \sum_{i=1}^{n} \left(1+\frac{6i}{n}+\frac{9i^2}{n^2}\right)\cdot\frac{3}{n} \\
&= \frac{3}{n}\left(\sum_{i=1}^{n} 1 + \frac{6}{n}\sum_{i=1}^{n} i + \frac{9}{n^2}\sum_{i=1}^{n} i^2\right) \\
&= \frac{3}{n}\sum_{i=1}^{n} 1 + \frac{18}{n^2}\sum_{i=1}^{n} i + \frac{27}{n^3}\sum_{i=1}^{n} i^2 \\
&= \frac{3}{n}(n) + \frac{18}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)
\end{aligned}$$

(d) Take the limit as $n \to \infty$

$$\begin{aligned}
\lim_{n\to\infty} \text{RHS} &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \text{(What you got in last problem)} \\
&= \lim_{n\to\infty} \left(\frac{3}{n}(n) + \frac{18}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right) \\
&= \lim_{n\to\infty} \left(\frac{3n}{n} + \frac{18n(n+1)}{2n^2} + \frac{27n(n+1)(2n+1)}{6n^3}\right) \\
&= \lim_{n\to\infty} \frac{3n}{n} + \lim_{n\to\infty} \frac{18n(n+1)}{2n^2} + \lim_{n\to\infty} \frac{27n(n+1)(2n+1)}{6n^3} \\
&= 3 + 9 + 9 = 21
\end{aligned}$$