Math 131 - April 18, 2016
Solutions

Warm-up Problems

1. Let $y = x^2$.
   Compute left and right hand sums for the area between $x = 1$ and $x = 4$.

   (a) $L_2 = 10.875$
   (b) $R_2 = 33.375$
   (c) $T_2 = 22.125$
   (d) $M_2 = 20.4375$
   (e) $L_3 = 14$
   (f) $R_3 = 29$
   (g) $T_3 = 21.5$
   (h) $M_3 = 20.75$
   (i) $L_6 = 17.375$
   (j) $R_6 = 24.875$
   (k) $T_6 = 21.125$
   (l) $M_6 = 20.9375$
   (m) $L_{100} = 20.77545$
   (n) $R_{100} = 21.22545$
   (o) $T_{100} = 21.00045$
   (p) $M_{100} = 20.999775$

   Exact: 21.0

Lecture Problems

2. Compute $\int_1^4 x^2\,dx$

   (a) $\Delta x = \frac{3}{n}$
   (b) $x_i = 1 + \frac{3i}{n}$
   (c) Continue with this and simplify (using what you found above)

$$\begin{aligned}
\text{RHS} &= \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} f(1+3i/n)\cdot\frac{3}{n} \\
&= \sum_{i=1}^{n} (1+3i/n)^2\cdot\frac{3}{n} \\
&= \sum_{i=1}^{n} \left(1+\frac{6i}{n}+\frac{9i^2}{n^2}\right)\cdot\frac{3}{n} \\
&= \frac{3}{n}\left(\sum_{i=1}^{n} 1+\frac{6}{n}\sum_{i=1}^{n} i+\frac{9}{n^2}\sum_{i=1}^{n} i^2\right) \\
&= \frac{3}{n}\sum_{i=1}^{n} 1+\frac{18}{n^2}\sum_{i=1}^{n} i+\frac{27}{n^3}\sum_{i=1}^{n} i^2 \\
&= \frac{3}{n}(n)+\frac{18}{n^2}\left(\frac{n(n+1)}{2}\right)+\frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)
\end{aligned}$$

(d) Take the limit as $n \to \infty$

$$\begin{aligned}
\lim_{n\to\infty} \text{RHS} &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} (\text{What you got in last problem}) \\
&= \lim_{n\to\infty} \left(\frac{3}{n}(n) + \frac{18}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right) \\
&= \lim_{n\to\infty} \left(\frac{3n}{n} + \frac{18n(n+1)}{2n^2} + \frac{27n(n+1)(2n+1)}{6n^3}\right) \\
&= \lim_{n\to\infty} \frac{3n}{n} + \lim_{n\to\infty} \frac{18n(n+1)}{2n^2} + \lim_{n\to\infty} \frac{27n(n+1)(2n+1)}{6n^3} \\
&= 3 + 9 + 9 = 21
\end{aligned}$$