Math 131 - April 11, 2016

Test Review Problems

1. True/False
   (a) $\lim_{x\to 2} \frac{x^3-4}{x+2} = 12$
   (b) If $f'(c) = 0$ then c is either a local maximum or a local minimum.
   (c) A function $f(x)$ can have infinitely many different functions $F(x)$ such that $F'(x) = f(x)$.
   (d) A differentiable function satisfying $f(-1) = f(1)$ and $f'(x) \geq 0$ for all x must be a constant function on the interval $[-1, 1]$.
   (e) There is a function with domain $\mathbb{R}$ satisfying the following: $f(x) < 0$ for all x, $f'(x) > 0$ for all x and $f''(x) > 0$ for all x.
2. Find the minimum value of $f(x) = (x-3)\sqrt{x}$ on $[0, 3]$
3. Find the maximum value of $f(x) = x^2/(1+x^2)$ on $[-1, 2]$.
4. Evaluate $\lim_{x\to 0^+} \sqrt{x}\ \ln x$.
5. Evaluate $\lim_{x\to 0} \ln(1+2x)/(e^{3x}-1)$
6. Find the absolute min of $f(x) = x^3 - 12x + 1$ on $[-3, 5]$.
7. For what values of a and b is $(1, 6)$ a point of inflection for the curve $y = x^3 + ax^2 + bx + 1$?
8. Let $f(x) = \ln(x^2 - 2x + 2)$. Graph, identify all max, min, inflections points and everything else important.
9. Find $\lim_{x\to\infty} \ln\left(\frac{x+1}{3x}\right)$
10. Find the equation of the slant asymptote for $y = \frac{3x^4-2x^3+x-5}{x+1}$.
11. Find the equation of the slant asymptote for $y = \frac{3x^4-2x^3+x-5}{x^2+1}$.
12. Find the equation of the slant asymptote for $y = \frac{3x^4-2x^3+x-5}{x^3+1}$.
13. Find the equation of the slant asymptote for $y = \frac{3x^4-2x^3+x-5}{x^4+1}$.
14. A farmer has 600 yards of fencing to build a rectangular pen. Some of the fencing will be used to contruct two interval divider fences, both parallel to the ends of the pen. What is the maximum area of such a pen?
15. Given $f'(x) = (x-1)/x^3$, where is f concave down?
16. Given $a > 1$ and $b > 1$, find $\lim_{x\to 0} \frac{a^x - b^x}{x}$
17. Find $\lim_{x\to\infty}(1 + b/x)^x$
18. Let $f(x) = xe^{-x}$. Find all extrema of f on the interval $[-2, 2]$.
19. Let $f(x) = \frac{1}{x^2+4}$ Find the critical points and inflection points of $f(x)$

20. A farmer builds a fence to enclose a rectangular field and he plans to put a fence down the middle of the field to separate the field into two equal portions. The fencing for the perimeter costs \$2/foot and the fencing for the divider costs \$4/foot. What dimensions should be farmer choose for the enclosure to maximize the area given that he has \$2000 to spend on fencing?

21. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost $\$10/ft^2$ and the material used to build the sides cost $\$6/ft^2$. If the box must have a volume of $50ft^3$, determine the dimensions that will minimize the cost to build the box.

22. $\lim_{x\to\infty} \frac{3^x}{x^2+x-1} =$

23. $\lim_{x\to 0} \frac{\sin x}{1-\cos x} =$

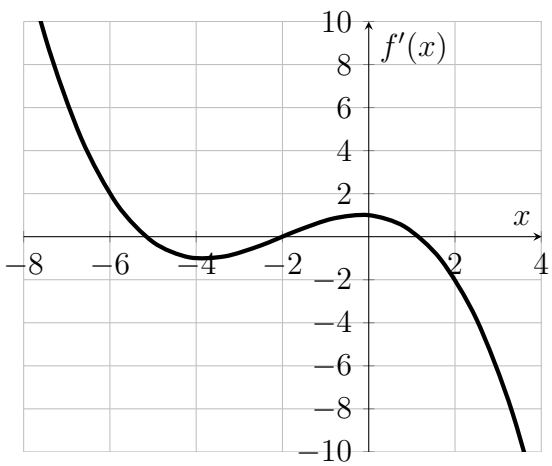
24. $\lim_{x\to 0} \sin x \ln x =$

25. $\lim_{x\to 0} x^{2x} =$

26. $\lim_{x\to 0} x^{x} =$

27. $\lim_{x\to 0} x^{1/x} =$

28. Given the graph of $f'(x)$ below, tell me everything you can about $f(x)$: max, mins, inflection points, critical points, etc.



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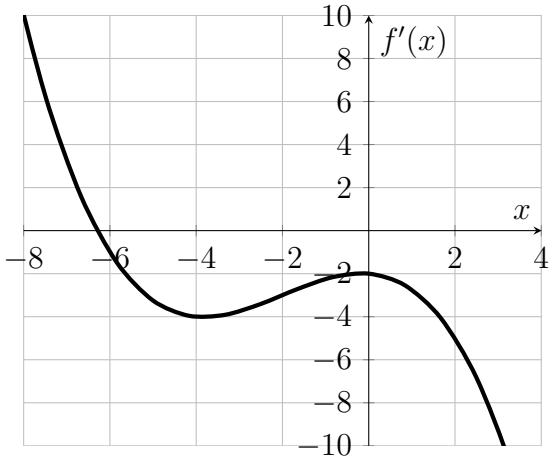

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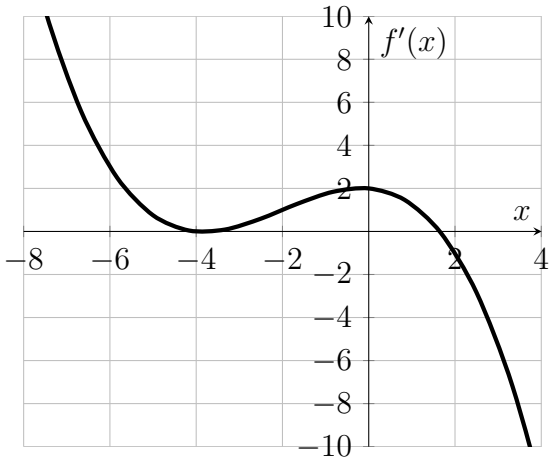


MORE!

31. Find the maximum value of $f(x) = x^2/(1 + x^2)$ on $[-1, 2]$.

32. Two numbers have a sum of 12. Find the maximum product of these numbers.

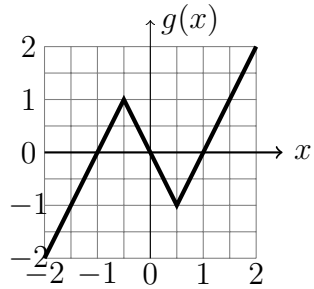
33. A rectangle has a vertex at the origin, base on the positive x-axis and a vertex in quadrant I on the line $x + 2y = 8$. Place this vertex on quadrant I so that the area of the rectangle can be maximized.

34. A rectangle has a vertex at the origin, base on the positive x-axis and a vertex in quadrant I on the line $x + 2y = 8$. Place this vertex on quadrant I so that the perimeter of the rectangle can be maximized.

35. given $x^3 + ax^2 + 3x + 1$ has an inflection point at $x = -2$, find a.

36. Let $f(x) = x(x + 1)(x - 1)$. Find the average slope over $[-1, 2]$. Find a point c in $(-1, 2)$ such that $f'(c)$ is equal to the average slope. If there is not such a point, explain why the Mean Value Theorem doesn't hold.

37. Let $g(x)$ be as below.



Find the average slope over $[-1, 2]$. Find a point c in $(-1, 2)$ such that $g'(c)$ is equal to the average slope. If there is not such a point, explain why the Mean Value Theorem doesn't hold.

38. $\lim_{x\to 0} x^{(x^x)} =$

39. $\lim_{x\to 0} x^{1/\ln(x)} =$

40. $\lim_{x\to 0} x^{\ln(x)} =$

41. Draw a graph of f with the following sign chart:

		-2	-1	0		5		8	
f''	$-$		$-$	$-$	$-$		$-$		$+$
f'	$+$		$+$	$+$	$-$		$-$		$-$

x

42. Draw a graph of f with the following sign chart:

		-2	-1	0		5		8	
f''	$+$		$-$	$-$	$+$		$-$		$+$
f'	$+$		$-$	$-$	$+$		$-$		$-$

x

43. Find the point on the line $x + y = 1$ that is closest to the point $(1, 5)$.

44. Find the slope and y-intercept of the line through the point $(-1, 5)$ that cuts off the least area from the second quadrant.

45. A woman is standing at a point on the shore of a circular lake with radius 2. She wants to arrive at the point diametrically opposite to her on the other side of the lake in the shortest possible time. She can walk at the rate of 10 mph and row a boat at 5 mph. What is the shortest amount of time it would take her to reach her desired point?

    Note: this is a very difficult problem. You should do what you can to try to find a good variable to use and then write a formula down for travel time.