Math 131 - April 6, 2016
Solutions

Warm-up Problems

1. Determine if there is a slant asymptote and find it if there is one:

   (a) $y = \frac{x^3-12x^2+38x-17}{x-7} = x^2 - 5x + 3 + \frac{4}{x-7}$
   Solution: No slant asymptote.

   (b) $y = \frac{x^3-12x^2+38x-17}{x^2-7} = x - 12 + \frac{45x-101}{x^2-7}$
   Solution: Slant asymptote: $y = x - 12$.

   (c) $y = \frac{x^3-12x^2+38x-17}{x^3-7} = 1 + \frac{-12x^2+38x-10}{x^3-7}$
   Solution: Horizontal asymptote: $y = 1$.

Lecture Problems

For the optimization problems below

- Identify what is being asked to optimize. Are you being asked to find a max or a min? (You probably want to draw a picture at this stage!)
- Find a function to represent what is to be optimized.
- Find the domain of your function.
- Optimize and solve the problem

2. Find 2 positive numbers whose sum is 50 and product is as large as possible.

   Solution: Maximize $P = xy$ subject to $x + y = 50$. Substitute to get $P = x(50 - x)$. Domain is $x \in [0, 50]$. Maximum of $P = 25^2$ occurs when $x = 25$

3. Find the point(s) on the curve $y = 25 - x^2/4$ closest to the origin.

   Solution: Minimize $D = x^2 + (25 - x^2/4)^2$. Domain is $x \in (-\infty, \infty)$. Min occurs when $x = \pm\sqrt{92}$.

4. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

   Solution: Maximize $A = xy$ subject to $2x + y = 2400$. Substitute and get $A = 2400x - 2x^2$. Domain is $x \in [0, 1200]$. Maximum is $720,000$ when $x = 600$ (and $y = 1200$).

5. A window is being built and the bottom is a rectangle and the top is a semicircle. If there is 12 m of framing materials what must the dimensions of the window be to let in the most light?

   Solution: Maximize area $A = 2hr + \frac{1}{2}\pi r^2$. Constraint on perimeter is $2h + 2r + \pi r = 12$. Substitute to get $A = 12r - (2 + \pi/2)r^2$. Domain $r \in [0, 12/(2 + \pi)]$.

   Maximum of $A \approx 10.08$ occurs when $r = 12/(4 + \pi)$.

6. A triangle has an angle θ and side lengths of 3 on either side of the angle θ. Find the value of θ so that the isosceles triangle will have the largest area.

   Solution: Let x be the other side of the triangle and y the height from the side x. So, we want to maximize $A = xy/2$. Use a little bit of trig to find x and y in terms of θ: $x = 6\sin(\theta/2)$ and $y = 3\cos(\theta/2)$. Domain is $\theta \in [0, \pi]$. Maximum occurs when $\theta = \pi/2$.