**Warm-up Problems**

1. Determine if there is a slant asymptote and find it if there is one:
   
   (a) \[ y = \frac{x^3 - 12x^2 + 38x - 17}{x^2 - 7} = x^2 - 5x + 3 + \frac{4}{x^2 - 7} \]
   
   **Solution:** No slant asymptote.

   (b) \[ y = \frac{x^3 - 12x^2 + 38x - 17}{x^2 - 7} = x - 12 + \frac{45x - 101}{x^2 - 7} \]
   
   **Solution:** Slant asymptote: \( y = x - 12 \).

   (c) \[ y = \frac{x^3 - 12x^2 + 38x - 17}{x^3 - 7} = 1 + \frac{-12x^2 + 38x - 10}{x^3 - 7} \]
   
   **Solution:** Horizontal asymptote: \( y = 1 \).

**Lecture Problems**

For the optimization problems below

- Identify what is being asked to optimize. Are you being asked to find a max or a min? (You probably want to draw a picture at this stage!)
- Find a function to represent what is to be optimized.
- Find the domain of your function.
- Optimize and solve the problem

2. Find 2 positive numbers whose sum is 50 and product is as large as possible.
   
   **Solution:** Maximize \( P = xy \) subject to \( x + y = 50 \). Substitute to get \( P = x(50 - x) \). Domain is \( x \in [0, 50] \). Maximum of \( P = 25^2 \) occurs when \( x = 25 \).

3. Find the point(s) on the curve \( y = 25 - x^2/4 \) closest to the origin.
   
   **Solution:** Minimize \( D = x^2 + (25 - x^2/4)^2 \). Domain is \( x \in (-\infty, \infty) \). Min occurs when \( x = \pm \sqrt{\frac{92}{4}} \).

4. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?
   
   **Solution:** Maximize \( A = xy \) subject to \( 2x + y = 2400 \). Substitute and get \( A = 2400x - 2x^2 \). Domain is \( x \in [0, 1200] \). Maximum is 720,000 when \( x = 600 \) (and \( y = 1200 \)).

5. A window is being built and the bottom is a rectangle and the top is a semicircle. If there is 12 m of framing materials what must the dimensions of the window be to let in the most light?
   
   **Solution:** Maximize area \( A = 2hr + \frac{1}{2} \pi r^2 \). Constraint on perimeter is \( 2h + 2r + \pi r = 12 \). Substitute to get \( A = 12r - (2 + \pi/2)r^2 \). Domain \( r \in [0, 12/(2 + \pi)] \).
   
   Maximum of \( A \approx 10.08 \) occurs when \( r = 12/(4 + \pi) \).
6. A triangle has an angle $\theta$ and side lengths of 3 on either side of the angle $\theta$. Find the value of $\theta$ so that the isosceles triangle will have the largest area.

**Solution:** Let $x$ be the other side of the triangle and $y$ the height from the side $x$. So, we want to maximize $A = xy/2$. Use a little bit of trig to find $x$ and $y$ in terms of $\theta$: $x = 6 \sin(\theta/2)$ and $y = 3 \cos(\theta/2)$. Domain is $\theta \in [0, \pi]$. Maximum occurs when $\theta = \pi/2$. 