Warm-up Problems

1. Draw a graph that matches the given data. Label all local max and mins and all inflections points.

<table>
<thead>
<tr>
<th>$f''(x)$</th>
<th></th>
<th>+</th>
<th></th>
<th>-</th>
<th>+</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'(x)$</td>
<td>+</td>
<td></td>
<td>-</td>
<td></td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>$x &lt; 3$</td>
<td>$3 &lt; x &lt; 4$</td>
<td>$4 &lt; x &lt; 5$</td>
<td>$5 &lt; x &lt; 6$</td>
<td>$x &gt; 6$</td>
</tr>
</tbody>
</table>

Lecture Problems

2. Suppose you are given:

\[
\begin{align*}
f(x) &= \frac{4x}{x^2 + 1} \\
f'(x) &= -\frac{4(x - 1)(x + 1)}{(x^2 + 1)^2} \\
f''(x) &= \frac{8x(x^2 - 3)}{(x^2 + 1)^3}
\end{align*}
\]

(a) Find all critical points of $f$. \textbf{Solution:} $x = 1, -1$. \((-1, -2) \text{ and } (1, 2)\).

(b) Find all possible inflection points of $f$. \textbf{Solution:} $x = \pm\sqrt{3}$: \((-\sqrt{3}, -\sqrt{3}) \text{ and } (\sqrt{3}, \sqrt{3})\)

(c) Find all intervals where $f$ is increasing and decreasing.

(d) Find all intervals where $f$ is concave up and down.

(e) Find all local max and mins of $f$.

(f) Find all inflection points of $f$.

(g) Draw a nice graph of $f$. 